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ABSTRACT

This dissertation studies three economic problems plagued by multiple equilibria. Indeterminacy of equilibrium outcome often poses a challenge in deriving robust predictions and policy guidance. The dissertation shows how the utilization of potential game theory can better deal with this challenge.

Chapter 1 studies a general contracting problem between one principal and multiple agents. The interdependence of agents' actions and payoffs creates a coordination problem among them, leading to multiple equilibria. In general, the principal's optimal contracting scheme varies with how one selects among equilibria. Nevertheless, for a large class of contracting models where agents' payoffs constitute a weighted potential game, I show that one contracting scheme is optimal for a large class of equilibrium selection criteria. This scheme ranks agents in increasing weight in the weighted potential game and induces them to accept their offers in a dominance-solvable way, starting from the first agent. I also apply the general results to networks and pure/impure public goods/bads.

Chapter 2 studies two-sided markets, where two groups of agents interact via platforms. These markets exhibit network effects, i.e., the value of joining a platform increases with the number of users, which in turn lead to multiple equilibria. I show that many two-sided market models are weighted potential games, enabling the selec-

tion among equilibria by potential maximization—a refinement of Nash equilibrium justified by many theoretical and experimental studies. Under potential maximization, platforms often charge the side deriving more network benefits and subsidize the other side. Therefore, profit-maximizing platforms are often designed to favor the money side much more than the subsidy side.

Chapter 3 studies markets with strong network effects. In these markets, firms compete for the adoption of all consumers rather than the marginal consumer. Therefore, the Spence distortion—a quality distortion driven by competition for the marginal consumer—should be absent, contradicting the findings in the network economics literature. This inconsistency stems from the choice of equilibrium selection criterion. I show that all popular selection criteria in that literature lead to Spence distortions, whereas potential maximization does not. Therefore, network market regulations based on Spence distortion arguments may be misguided.

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Chapter 1

Weight-Ranked Divide-and-Conquer Contracts

1.1 Introduction

Many contracting situations involve multiple agents, and in most of these situations, an agent's payoff depends on other agents' actions. For example, the value of joining a platform increases with the number of users; the return from an investment is affected by others' investment decisions; the incentive to work changes with co-workers' efforts. A natural question arises: how does the principal's optimal contracting scheme take into account these (potentially very complex) interactions among agents? Moreover, agents' strategic interactions often generate multiple equilibria. All the above examples may have (at least) a high- and a low-participation/investment/effort equilibrium. The principal's payoff typically differs across equilibria. This raises a more fundamental problem: how should we define the *optimality* of a contracting scheme when there are multiple equilibria? Ultimately, what contracts should the principal offer when there are multiple agents?

The most common approach to deal with the fundamental problem is to specify an equilibrium selection criterion (or, more generally, an implementation requirement)[1] and get rid of multiple equilibria. However, this approach does not fully resolve the problem because it replaces the issue of multiple equilibria with the issue of multiple equilibrium selection criteria. To illustrate, the optimal contracts for the best-case scenario are likely rejected by agents in less favorable scenarios. On the other hand, the optimal contracts for

[1] As I will formally show on p. 10, any equilibrium selection criterion can be expressed as an implementation requirement, but not vice versa. For example, the best (worst) equilibrium selection criterion for the principal is equivalent to partial (robust) implementation, but any requirement stronger than robust implementation (e.g., unique, dominance-solvable, or dominant-strategy implementation) is not an equilibrium selection criterion.

the worst-case scenario likely forgo huge profits in more favorable scenarios. If the principal (or we, as researchers) does not know the underlying equilibrium selection criterion, can we still recommend (or reasonably predict) what contracts the principal should (or would) offer?

This paper shows that the answer to the above question is “yes” for a large class of contracting models with complete information. The timing is standard: the principal offers each agent a menu of publicly observable bilateral[2] contracts in stage 1, and each agent simultaneously chooses a contract (or rejects all contracts) in stage 2. Each agent's set of possible actions can be any compact set. Regarding their interactions, some agents' actions can be strategic complements while some others can be strategic substitutes. The principal can be self-interested or benevolent. The only key assumption, which is a technical assumption, is that agents' payoffs constitute a *weighted potential game*. The solution concept is subgame-perfect Nash equilibrium with an equilibrium selection criterion. One equilibrium selection criterion is said to be *more pessimistic* than the other if, in every stage-2 subgame, the selected equilibrium gives the principal a weakly lower payoff. As a technical tool, an equilibrium selection criterion, *potential maximization*, plays a crucial role in the analysis.

The main result of this paper is that under any equilibrium selection criterion that is more pessimistic than potential maximization, the principal's optimal contracting scheme is to offer *weight-ranked divide-and-conquer* ($\mathbf{w}$-DC) *contracts*. The $\mathbf{w}$-DC contracts rank agents in increasing order of their *weights* in the weighted potential game and offer each agent *one*[3] contract asking him to take a specified action. The associated contract prices or subsidies are set in a way that the first agent has a weakly dominant strategy to accept the offer; given the first agent accepts the offer, the second agent has an (iterated) weakly

[2] As pointed out by the literature (e.g., Bernstein and Winter 2012; Halac et al. 2020), the principal can only rely on bilateral contracts in many real-world contracting situations. If the principal is allowed to offer multilateral contracts (i.e., contracts that can condition on others' actions), she can easily induce a unique equilibrium that fully extracts all agents' surplus in most such models.

[3] Note that the principal can offer each agent a menu of contracts, but it turns out she only needs to offer one contract to each agent in this optimal contracting scheme.

dominant strategy to accept the offer as well, and so on. Thus, the **w**-DC contracts induce all agents to accept their offers as a dominance-solvable equilibrium in a particular order. In addition, I show that the **w**-DC contracts may be suboptimal if the underlying equilibrium selection criterion is *not more pessimistic* than potential maximization. Hence, I have identified the complete set of equilibrium selection criteria for which the principal always offers the **w**-DC contracts. I further extend the main result by showing that the **w**-DC contracts are optimal for a large class of implementation requirements, which includes robust, unique, and dominance-solvable implementation.

Section 1.4 of this paper applies the general results to three special cases: networks, public goods/bads (henceforth goods for simplicity), and impure public goods. In the undirected network model, agents can be heterogeneous in many aspects, including their (i) sets of connected agents, (ii) valuations of network benefits, and (iii) importance to their connected agents regarding network benefits. The **w**-DC contracts rank agents in increasing valuation-to-importance ratio. Contrary to the conventional wisdom in the economics of networks literature (see, e.g., Section 7 of Jackson et al. 2017 for a survey) that the principal should offer more favorable contracts to agents with high centrality (i.e., at central positions in the network), the network structure plays no role in agents' ranking.[4] I further derive a natural network formation process and show that under this process, highly connected agents are those with high valuations and those with either high or low importance.

In the public good model, agents can be heterogeneous in many aspects, including their (i) valuations of the public good and (ii) importance of their contributions to the public good. In contrast to the network model, the **w**-DC contracts rank agents in increasing valuation regardless of their importance. As the impure public good model clarifies, the reason for these opposing results is that the public good is non-excludable whereas the "network good" is excludable. In that model, the **w**-DC contracts always prioritize agents

[4] As I will explain more on p. 15, higher-ranked agents receive better prices. In particular, the first agent has a dominant strategy to accept the offer, whereas the last agent is indifferent between accepting and rejecting the offer in equilibrium.

with low valuations; they also prioritize those with high importance if and only if the good is sufficiently excludable.

This paper makes two general contributions. First, it extends our understanding of multi-agent contracting. Various contracting schemes are derived under different implementation requirements in the literature. For example, the seminal papers by Segal (1999, 2003) study the same model and derive very different contracting schemes under partial and unique implementation respectively. This paper shows that one contracting scheme—the **w**-DC contracts—is particularly robust because it is optimal for a large class of equilibrium selection criteria and implementation requirements. This result helps us make better predictions and policy advice on multi-agent contracting problems especially when we, as researchers, do not know the underlying equilibrium selection criterion or what implementation requirement the principal aims to meet.

In the literature, specific divide-and-conquer (DC) contracts are derived (e.g., Segal 2003; Winter 2004; Bernstein and Winter 2012; Halac et al. 2020) under (i) binary/one-dimensional actions for agents, (ii) strategic complementarities among all agents, and (iii) the requirement of unique implementation. This paper uncovers the generality and robustness of the DC contracts by relaxing all three restrictions substantially and deriving the general form of the DC contracts. Furthermore, this paper is the first to derive the optimal ranking for the DC contracts in a general setting and, thus, fully characterize the optimal contracting scheme: the **w**-DC contracts.

Second, the paper advances the analysis of multi-agent contracting problems. Although its primary focus is the **w**-DC contracts, the general framework and tools developed are applicable to all such problems. In particular, the novel interaction structure among agents, which is derived from two binary relations, is considerably more flexible than the usual strategic complementarity/substitutability structure, enabling us to study a wider range of contracting situations. In addition, a methodological contribution of this paper is to incorporate potential game theory into multi-agent contracting. The concept of potential games was introduced by Rosenthal (1973) and formalized by Monderer and Shapley (1996).

Potential maximization refines Nash equilibrium in (weighted) potential games.[5] I will explain both concepts in Section 1.2. As Section 1.4 reveals, agents' payoffs constitute a weighted potential game in many contracting models. By exploiting this useful property, one may be able to derive stronger results as this paper does.

Beyond the above general contributions, Section 1.4 contributes to the literature on the economics of networks, public goods, and impure public goods.[6] One contribution common to all three strands of literature is to show the optimality and robustness of the corresponding $\mathbf{w}$-DC contracts in each of the environments. A paper related to all three applications is Sakovics and Steiner (2012). They analyze a binary-action complete network under *global-game selection* (which is equivalent to potential maximization; see footnote 5) and show that the principal ranks agents in increasing valuation-to-importance ratio and then offers divide-and-conquer contracts. My network model uncovers the robustness of their main finding to (i) more general actions, (ii) arbitrary undirected network structures, and (iii) all equilibrium selection criteria more pessimistic than global-game selection. However, my (impure) public good model reveals that their finding is not robust to (partially) non-excludable externalities. Hence, my results urge caution in applying their finding to their leading applications—economic development and financial fragility—which are largely public goods/bads in nature or at least partially non-excludable. My impure public good model provides refined guidance on vertical/sectoral industrial policies (a.k.a. "picking winners") for the former and financial policies for the latter.

[5] Although inconsequential to my main result, this refinement is justified by many theoretical and experimental studies; see Chan (2019, Related Literature) for a summary of established justifications. In particular, it coincides with global-game selection in supermodular weighted potential games (Frankel at el. 2003) and risk dominance in two-agent two-action games.

[6] For the recently growing literature on contracting in networks, see, for example, Belhaj and Deroian (2019), Jadbabaie and Kakhbod (2019), Bloch and Shabayek (2020), Shi and Xing (2020), and Zhang and Chen (2020). See Bergstrom et al. (1986) and Cornes and Sandler (1984, 1994) for the seminal works on public goods and impure public goods respectively.

1.2 Model

A principal ("she") contracts with N agents ("he"). With a slight abuse of notation, let $N \equiv \{1, \ldots, N\}$ also denote the set of agents. Let $x_i \in X_i$ denote the action of agent $i \in N$, where X_i can be any compact set with $o_i \in X_i$ denoting the outside option of rejecting the principal's offers. Let $\mathbf{x} \in X \equiv \prod_i X_i$ denote agents' action profile, and $\mathbf{x}_{-i} \in X_{-i} \equiv \prod_{j \neq i} X_j$ and $\mathbf{x}_{-ij} \in X_{-ij} \equiv \prod_{k \notin \{i,j\}} X_k$ are defined in the usual way. The game has two stages. In stage 1, the principal sets a price function $p_i \in P_i \equiv \{p_i : X_i \to \mathbb{R} | p_i(o_i) = 0\}$ for each agent i. This is equivalent to offering each agent a menu of bilateral contracts $(x_i, p_i(x_i))$ given that she can always prevent an agent from taking a certain action $x_i \in X_i \backslash \{o_i\}$ by charging an arbitrarily high price $p_i(x_i)$ for that action. Let $\mathbf{p} \in P \equiv \prod_i P_i$ denote the menu profile offered to agents. In stage 2, all agents observe $\mathbf{p}$ and simultaneously choose from $\Delta(X_i)$, i.e., mixed strategies are allowed.[7] Each agent i's payoff is linear in money, i.e., $u_i(\mathbf{x}) - p_i(x_i)$, where the continuous function $u_i : X \to \mathbb{R}$ measures his intrinsic utility. The principal's payoff is $U(\mathbf{x}, \sum_i p_i(x_i))$, where the upper semicontinuous function $U : X \times \mathbb{R} \to \mathbb{R}$ is weakly increasing in her total revenue $\sum_i p_i(x_i)$. The function U is sufficiently general to represent a self-interested (e.g., $U = \sum_i p_i(x_i)$) or benevolent (e.g., $U = \sum_i u_i(\mathbf{x})$) principal. All players are expected utility maximizers.

The results in Section 1.3 hold for all agents' (intrinsic) utilities $\mathbf{u} \equiv (u_i)_i$ satisfying the following three assumptions.

Assumption 1 (Weighted potential game) There exists a (weight) vector $\mathbf{w} \equiv (w_i)_i \in \mathbb{R}^N_{++}$ and a (potential) function $\Phi : X \to \mathbb{R}$ such that for all $i \in N$,

$$u_i(x_i, \mathbf{x}_{-i}) - u_i(x_i', \mathbf{x}_{-i}) = w_i[\Phi(x_i, \mathbf{x}_{-i}) - \Phi(x_i', \mathbf{x}_{-i})] \quad \text{for all } x_i, x_i' \in X_i \text{ and } \mathbf{x}_{-i} \in X_{-i}. \tag{1.1}$$

Assumption 1 (henceforth A1; similarly for A2 and A3) states that agents' utilities $\mathbf{u}$

[7] Most of this literature does not consider mixed strategies, but this is with loss of generality. For example, in a subgame, a mixed-strategy equilibrium may be the unique equilibrium giving the principal the highest expected payoff. Therefore, if the principal can select her most preferred equilibrium as in Segal (1999), she will select the mixed-strategy equilibrium.

constitute a *weighted potential game*. Verbally, there exists a real-valued function Φ defined on the set of action profiles such that the change in any agent's utility by unilaterally switching actions is proportional (with proportion w_i for agent i) to the corresponding change in Φ. Thus, Φ summarizes all agents' strategic considerations. Observe that (1.1) holds if and only if there exists a (pure externality) function $\xi_i : X_{-i} \to \mathbb{R}$ such that

$$u_i(\mathbf{x}) = w_i\Phi(\mathbf{x}) + \xi_i(\mathbf{x}_{-i}) \quad \text{for all } \mathbf{x} \in X.^8 \tag{1.2}$$

Many contracting models satisfy A1. For example, let X_i be any compact subset of $\mathbb{R}_+$ with $o_i = 0 \in X_i$ (e.g., $X_i = \{0, 1\}$ or $X_i = [0, \bar{x}_i]$) and u_i take the following form:

$$u_i(\mathbf{x}) = b_i(x_i) + v_i x_i \sum_{j \in E_i} \theta_j x_j, \tag{1.3}$$

where $b_i : X_i \to \mathbb{R}$ measures his stand-alone benefit/cost, $E_i \subseteq N\backslash\{i\}$ is the set of agents interacting with agent i (interactions are two-way, i.e., $j \in E_i$ iff $i \in E_j$), $v_i \in \mathbb{R}_{++}$ measures his valuation of interaction benefits, and $\theta_j \in \mathbb{R}_{++}$ measures the relative importance of agent j's actions to his interacting agents. Agents can differ in five dimensions $(X_i, b_i, E_i, v_i, \theta_i)$ in this general example, which in turn covers a wide variety of contracting situations. Section 1.4.1 analyzes this example and (Lemma 4) shows that it satisfies A1 (and A2 and A3). Sections 1.4.2 and 1.4.3 study two other examples that also satisfy A1–A3.

In order to state the other two assumptions on agents' utilities $\mathbf{u}$, I first define two binary relations C and S between any two distinct agents' action sets X_j and X_i.

Definition 1 The expression $x_j C x_i$ ($x_j S x_i$) stands for

$$u_i(x_i, x_j, \mathbf{x}_{-ij}) - u_i(o_i, x_j, \mathbf{x}_{-ij}) \geq (\leq)\ u_i(x_i, o_j, \mathbf{x}_{-ij}) - u_i(o_i, o_j, \mathbf{x}_{-ij}) \quad \forall \mathbf{x}_{-ij} \in X_{-ij}. \tag{1.4}$$

In words, $x_j C x_i$ ($x_j S x_i$) means x_j always strategically complements (substitutes) x_i

[8] The "if" part is trivial. For the "only if" part, the function $\xi_i(\mathbf{x}_{-i}) \equiv u_i(\mathbf{x}) - w_i\Phi(\mathbf{x})$ is well defined because, by (1.1), $u_i(x_i, \mathbf{x}_{-i}) - w_i\Phi(x_i, \mathbf{x}_{-i}) = u_i(x_i', \mathbf{x}_{-i}) - w_i\Phi(x_i', \mathbf{x}_{-i})$ for all $x_i, x_i' \in X_i$.

relative to the outside option. The second assumption is stated as follows.

Assumption 2 (Sign independence of others' actions) For each $\mathbf{x} \in X$ and distinct $i, j \in N$, $x_j C x_i$ or $x_j S x_i$.

To better understand A2, consider a scenario in which agents i and j only choose between a particular action (say, x_i for i and x_j for j) and the outside option, and all other agents choose the outside option. In this scenario, x_j either strategically complements or substitutes x_i because $u_i(x_i, x_j, \mathbf{o}_{-ij}) - u_i(o_i, x_j, \mathbf{o}_{-ij})$ is either greater or less than $u_i(x_i, o_j, \mathbf{o}_{-ij}) - u_i(o_i, o_j, \mathbf{o}_{-ij})$. A2 implies that if x_j strategically complements (substitutes) x_i in this scenario, then x_j always strategically complements (substitutes) x_i regardless of others' actions $\mathbf{x}_{-ij}$.

Observe from (1.1) and (1.4) that A1 implies C and S are symmetric, i.e., $x_j C x_i$ ($x_j S x_i$) if and only if $x_i C x_j$ ($x_i S x_j$).[9] In other words, any two agents' actions either strategically complement or substitute each other relative to the outside option. I write $x_j \bar{C} x_i$ if $x_j C x_i$ but not $x_j S x_i$. Clearly, $\bar{C}$ is also symmetric. The last assumption is stated as follows.

Assumption 3 (Weak transitivity for C) For each $\mathbf{x} \in X$ and distinct $i_1, i_2, \ldots, i_n \in N$ $(n \leq N)$, if $x_{i_1} \bar{C} x_{i_2} \bar{C} \cdots \bar{C} x_{i_n}$ then $x_{i_1} C x_{i_n}$.

Observe that A3 is weaker than assuming C ($\bar{C}$) is transitive, which replaces all $\bar{C}$ (C) in A3 with C ($\bar{C}$). A2 and A3 are rather weak. They are vacuous if there are only two agents. For more agents, they impose no restrictions on any two actions $x_i, x_i' \in X_i \backslash \{o_i\}$ from the same agent. In particular, they allow $x_j C x_i$ for some x_i but $x_j S x_i'$ for some other x_i'. Furthermore, even if A2 and A3 are strengthened to $x_j C x_i$ (similarly for $x_j S x_i$) for all $i, j \in N$ and $\mathbf{x} \in X$, this is still much weaker than the following strategic complementarity (similarly for substitutability) assumption, which is imposed by most of the literature and satisfied by the previous example (1.3).

[9] To see this more clearly, by (1.1), $u_i(x_i, x_j, \mathbf{x}_{-ij}) - u_i(o_i, x_j, \mathbf{x}_{-ij}) \geq u_i(x_i, o_j, \mathbf{x}_{-ij}) - u_i(o_i, o_j, \mathbf{x}_{-ij})$ iff $\Phi(x_i, x_j, \mathbf{x}_{-ij}) - \Phi(o_i, x_j, \mathbf{x}_{-ij}) \geq \Phi(x_i, o_j, \mathbf{x}_{-ij}) - \Phi(o_i, o_j, \mathbf{x}_{-ij})$ iff $u_j(x_i, x_j, \mathbf{x}_{-ij}) - u_j(x_i, o_j, \mathbf{x}_{-ij}) \geq u_j(o_i, x_j, \mathbf{x}_{-ij}) - u_j(o_i, o_j, \mathbf{x}_{-ij})$.

Condition 1 (Strategic complementarities) For all $i \in N$, $o_i = 0 \in X_i \subseteq \mathbb{R}_+$ and for all $x_i, x_i' \in X_i$ with $x_i > x_i'$, $u_i(x_i, \mathbf{x}_{-i}) - u_i(x_i', \mathbf{x}_{-i})$ is weakly increasing in $\mathbf{x}_{-i} \in X_{-i}$.

Observe that Condition 1 (henceforth C1) restricts agents' actions to a single dimension and imposes restrictions on every two pairs of actions (x_i, x_i') and (x_j, x_j') from any two agents. Unlike A2 and A3, C1 is far from vacuous when $N = 2$. Therefore, the extra flexibility of A2 and A3 enables us to study many more contracting situations. For example, in the context of public good provision, building a public facility involves careful planning, huge capital, on-site construction, and many other actions. These actions differ in nature and therefore cannot be meaningfully compared along a single dimension. Furthermore, actions of the same nature may be strategic substitutes (e.g., the provider may need only one good planner) whereas those of different nature may be strategic complements. Section 1.4.2 formalizes this example. Note that under A1–A3, each action profile $\mathbf{x} \in X$ can be partitioned into several groups, in which $x_j C x_i$ if x_j and x_i belong to the same group and $x_j S x_i$ otherwise.[10] And A1–A3 allow different action profiles to have different partitions.

1.2.1 Solution Concept

Recall that each menu profile $\mathbf{p} \in P$ leads to a different subgame in stage 2, and some subgames have multiple equilibria. Naturally, the solution concept is subgame-perfect Nash equilibrium with an equilibrium selection criterion. However, instead of imposing a specific equilibrium selection criterion as most of the literature does, I derive results that are relevant to *every* equilibrium selection criterion. To state the main result (Theorem 1) in Section 1.3, I need to define two notions. The first notion compares two equilibrium selection criteria.

Definition 2 One equilibrium selection criterion is *more pessimistic* than the other if, in every subgame, the equilibrium selected by the former gives the principal a weakly lower expected payoff than that by the latter.

[10] See the proof of Proposition 1 (p. 71) for how to partition each action profile. Bernstein and Winter (2012, Section 3.D) study a similar interaction structure in their binary-action model.

Observe from this definition that (i) any equilibrium selection criterion is more pessimistic than itself and (ii) the most pessimistic (optimistic) equilibrium selection criterion is the one that always selects the worst (best) equilibrium for the principal.

The second notion is an equilibrium selection criterion originated from potential game theory. For a weighted potential game, it is known that the maximizer of the potential function exists, is generically unique, and is a (generically pure-strategy) Nash equilibrium.[11] Furthermore, as I will prove shortly, every subgame is a weighted potential game as long as A1 holds. Hence, a well-defined equilibrium selection criterion is to select the potential maximizer in every subgame, and it is called *potential maximization*.

As we will see, the proof of Theorem 1 consists of three steps. First, I derive the equilibrium outcome under potential maximization. Precisely, I show that *weight-ranked divide-and-conquer* ($\mathbf{w}$-DC) *contracts* are optimal to the principal under potential maximization. This is the most difficult step, but it is only an intermediate step. Because after that, I show that the $\mathbf{w}$-DC contracts remain optimal for all equilibrium selection criteria that are more pessimistic than potential maximization. Lastly, I show that the $\mathbf{w}$-DC contracts may be suboptimal if the underlying equilibrium selection criterion is *not more pessimistic* than potential maximization. By combining the latter two results, I have identified the entire set of equilibrium selection criteria for which the $\mathbf{w}$-DC contracts are always optimal.

Before proceeding, we need to take care of two technical issues. For expositional convenience, I formally define an equilibrium selection criterion as a function $f : P \to \Delta(X)$ where $f(\mathbf{p})$ is a Nash equilibrium of the subgame with $\mathbf{p}$ set by the principal in stage 1. With a slight abuse of notation, let $F(\mathbf{x}) \equiv \{\mathbf{p} \in P | f(\mathbf{p}) = \mathbf{x}\}$ denote the set of

[11] Given X is compact and all the u_i functions (and thus the potential function Φ) are continuous, the potential maximizer exists by the extreme value theorem. See p. 78 for generic uniqueness of the potential maximizer. The potential maximizer is a Nash equilibrium: if someone deviates from the potential maximizer, the potential will decrease, and, by (1.1), the deviator will have a lower payoff. The potential of a mixed-strategy equilibrium is a convex combination of the potentials defined on the set of pure-strategy action profiles. Therefore, a mixed-strategy equilibrium is a potential maximizer only when all the respective pure-strategy action profiles are also potential maximizers; this is highly non-generic. For more interpretations of weighted potential games, see Chan (2019, Section 2.3).

menu profiles implementing $\mathbf{x} \in \Delta(X)$ under f. Thus, an equilibrium selection criterion f is expressed and henceforth interpreted as an *implementation requirement*, which is fully characterized by $F \equiv (F(\mathbf{x}))_{\mathbf{x}\in\Delta(X)}$. The first issue is that an optimal contracting scheme may not exist because $F(\mathbf{x})$ is not always closed.[12] To guarantee existence, I slightly relax each implementation requirement F by enlarging $F(\mathbf{x})$ to its closure for all $\mathbf{x} \in \Delta(X)$. The second issue is that potential maximization fails to select a unique equilibrium when there are multiple potential maximizers in a non-generic subgame. Nevertheless, the above enlargement already solves this issue. Precisely, after the enlargement, the principal can select among potential maximizers.

1.3 Analysis

The analysis before Proposition 1 relies only on A1; the subsequent analysis relies on all three assumptions. Detailed interpretation of the results is deferred to Section 1.3.1. As mentioned, the first step is to analyze the model under potential maximization. To do so, first I need to show that as long as agents' utilities $\mathbf{u}$ constitute a weighted potential game, any subgame with an arbitrary menu profile $\mathbf{p} \in P$ offered by the principal is also a weighted potential game. All proofs are in Appendix A.

Lemma 1 *Every subgame is a weighted potential game with the same weight vector* $\mathbf{w}$ *given in A1 and the potential function*

$$\Phi_{\mathbf{p}}(\mathbf{x}) = \Phi(\mathbf{x}) - \sum_i \frac{p_i(x_i)}{w_i}. \tag{1.5}$$

Under potential maximization, the principal's problem can be formulated as the following two-step optimization problem. In step 1, given a target action profile $\hat{\mathbf{x}} \in X$,[13]

[12] To illustrate, consider a one-agent example with $X_1 = \{o_1 = 0, 1\}$, $u_1(0) = 0$, $u_1(1) = 1$, and $U(x_1, p_1(x_1)) = p_1(x_1)$. Multiple equilibria exist only when $p_1(1) = 1$. If f selects $x_1 = 0$ when $p_1(1) = 1$ (and thus $F(1) = \{p_1 \in P_1 | p_1(1) < 1\}$, which is not closed), then an optimal contracting scheme does not exist.

[13] In the highly non-generic case where a mixed-strategy equilibrium is a potential maximizer (see footnote 11), the principal can select among all the respective pure-strategy potential maximizers. Given her expected payoff at the mixed-strategy potential maximizer is a convex combination of her payoffs at the respective pure-strategy potential maximizers, she can, without loss of optimality, neglect the mixed-strategy potential maximizer and select among the respective pure-strategy potential maximizers.

she chooses the optimal menu profile $\mathbf{p}^* \in P$ such that $\hat{\mathbf{x}}$ is a potential maximizer in the subgame, i.e.,

$$\max_{\mathbf{p}\in P} U(\hat{\mathbf{x}}, \sum_i p_i(\hat{x}_i)) \quad \text{s.t.} \quad \Phi_{\mathbf{p}}(\hat{\mathbf{x}}) \geq \Phi_{\mathbf{p}}(\mathbf{x}) \quad \text{for all } \mathbf{x} \in X. \tag{1.6}$$

In step 2, she chooses the optimal action profile $\mathbf{x}^* \in X$, i.e.,

$$\max_{\mathbf{x}\in X} U(\mathbf{x}, \sum_i p_i^*(x_i)). \tag{1.7}$$

The main analysis is on the step-1 problem (1.6). Observe that given a fixed $\hat{\mathbf{x}}$, the principal's objective is to maximize her total revenue $\sum_i p_i(\hat{x}_i)$. Also, the constraints can be simplified with (1.5). Thus, (1.6) is simplified to

$$\max_{\mathbf{p}\in P} \sum_i p_i(\hat{x}_i) \quad \text{s.t.} \quad \sum_i \frac{p_i(\hat{x}_i) - p_i(x_i)}{w_i} \leq \Phi(\hat{\mathbf{x}}) - \Phi(\mathbf{x}) \quad \text{for all } \mathbf{x} \in X. \tag{1.8}$$

Now observe that charging arbitrarily high prices $p_i(x_i)$ for all $x_i \notin \{o_i, \hat{x}_i\}$ of every agent $i \in N$ relaxes all constraints involving $x_i \notin \{o_i, \hat{x}_i\}$ and has no impact on her total revenue. Therefore, we have the following lemma.

Lemma 2 *Under potential maximization, the principal can restrict herself to offering (at most) one contract to each agent without loss of optimality.*

Under this restriction, agent i only chooses from $\{o_i, \hat{x}_i\}$ if $\hat{x}_i \neq o_i$ and chooses the outside option otherwise. Denote $\hat{N} \equiv \{i \in N | \hat{x}_i \neq o_i\}$ as the set of agents whom the principal wants to contract with, $\hat{p}_i \equiv p_i(\hat{x}_i)$ as the respective contract price, and $\hat{\mathbf{p}} \equiv (\hat{p}_i)_{i\in\hat{N}}$ as the price vector. The step-1 problem (1.8) is further simplified as follows.

Lemma 3 *For any target action profile $\hat{\mathbf{x}} \in X$, the principal's optimal contracts under potential maximization solve the following linear program:*

$$\max_{\hat{\mathbf{p}}\in\mathbb{R}^{|\hat{N}|}} \sum_{i\in\hat{N}} \hat{p}_i \quad \textit{s.t.} \quad \sum_{i\in\hat{N}:x_i=o_i} \frac{\hat{p}_i}{w_i} \leq \Phi(\hat{\mathbf{x}}) - \Phi(\mathbf{x}) \quad \textit{for all } \mathbf{x} \in \prod_i \{o_i, \hat{x}_i\}. \tag{1.9}$$

Without A2 or A3, the above finite linear program is still computationally very tractable. With A2 and A3, the optimal contracts have closed forms as characterized by the following proposition.

Proposition 1 *Relabel the agents such that $w_1 \leq \cdots \leq w_N$. For any target action profile $\hat{\mathbf{x}} \in X$, the principal's optimal contracts under potential maximization are*

$$\hat{p}_i^* = u_i(\hat{x}_1, \ldots, \hat{x}_{i-1}, \hat{x}_i, \hat{y}_{i+1}, \ldots, \hat{y}_N) - u_i(\hat{x}_1, \ldots, \hat{x}_{i-1}, o_i, \hat{y}_{i+1}, \ldots, \hat{y}_N) \quad \text{for all } i \in N, \tag{1.10}$$

where $\hat{y}_j = o_j$ if $\hat{x}_j C \hat{x}_i$ and $\hat{y}_j = \hat{x}_j$ otherwise.[14] *If $w_1 < \cdots < w_N$, the above contracts are the unique optimal contracts.*

I call the above contracts (1.10) *weight-ranked divide-and-conquer* (**w**-DC) *contracts.* To better understand the **w**-DC contracts, first consider the following special case where all target actions are strategic complements.

Corollary 1 *If $\hat{x}_j C \hat{x}_i$ for all $i, j \in N$, the $\mathbf{w}$-DC contracts reduce to*

$$\hat{p}_i^* = u_i(\hat{x}_1, \ldots, \hat{x}_i, o_{i+1}, \ldots, o_N) - u_i(\hat{x}_1, \ldots, \hat{x}_{i-1}, o_i, \ldots, o_N) \quad \text{for all } i \in N. \tag{1.11}$$

I call (1.11) *weight-ranked simple divide-and-conquer* (**w**-SDC) *contracts.* In words, the **w**-SDC contracts rank agents in increasing order of weight w_i and offer each agent a price that would make him indifferent between accepting and rejecting the offer if all agents who precede him in the ranking accept their offers and all subsequent agents reject their offers. Given that $\hat{x}_j C \hat{x}_i$ for all $i, j \in N$, the first agent has a weakly dominant strategy to accept the offer.[15] And given the first agent accepts the offer, the second agent also has an (iterated) weakly dominant strategy to accept the offer, and so on. Thus, the **w**-SDC contracts in fact implement $\hat{\mathbf{x}}$ as a dominance-solvable equilibrium in this special case.

In the general case, the **w**-DC contracts refine the **w**-SDC contracts by compensating for the strategic substitutabilities from all subsequent agents. By the same token, the

[14] For notational convenience, the optimal contracts also include agents with $\hat{x}_i = o_i$. For these agents, we have $\hat{p}_i^* = 0$, which coincides with the requirement that $p_i(o_i) = 0$.

[15] Formally, we have $u_1(\hat{x}_1, \mathbf{x}_{-1}) - \hat{p}_1^* \geq u_1(o_1, \mathbf{x}_{-1})$ for all $\mathbf{x}_{-1} \in \prod_{i \neq 1}\{o_i, \hat{x}_i\}$. This is because $\hat{p}_1^* = u_1(\hat{x}_1, \mathbf{o}_{-1}) - u_1(\mathbf{o})$ by (1.11) and $\hat{x}_i C \hat{x}_1$ for all $i \neq 1$ implies $u_1(\hat{x}_1, \mathbf{x}_{-1}) - u_1(o_1, \mathbf{x}_{-1}) \geq u_1(\hat{x}_1, \mathbf{o}_{-1}) - u_1(\mathbf{o})$ for all $\mathbf{x}_{-1} \in \prod_{i \neq 1}\{o_i, \hat{x}_i\}$.

$\mathbf{w}$-DC contracts implement $\hat{\mathbf{x}}$ as a dominance-solvable equilibrium in this case.[16] Note that what I have just described is a property of the $\mathbf{w}$-DC contracts, i.e., it is unrelated to the underlying equilibrium selection criterion. In other words, under any equilibrium selection criterion, the principal can always offer the $\mathbf{w}$-DC contracts and implement $\hat{\mathbf{x}}$ in the same dominance-solvable way.[17] In addition, under any equilibrium selection criterion more pessimistic than potential maximization, the principal, by Definition 2, cannot do better than she does under potential maximization. Therefore, the $\mathbf{w}$-DC contracts remain optimal for all these equilibrium selection criteria. In the proof of Theorem 1, I construct a simple two-agent two-action example, in which the $\mathbf{w}$-DC contracts are suboptimal for all equilibrium selection criteria that are not more pessimistic than potential maximization. This completes the proof of the main result of this paper.

Theorem 1 *The* $\mathbf{w}$*-DC contracts are optimal for all games if and only if the underlying equilibrium selection criterion is more pessimistic than potential maximization.*

I now restrict attention to situations in which the principal always offers the $\mathbf{w}$-DC contracts and analyze her step-2 problem (1.7). From (1.10), her equilibrium payoff given a target action profile $\hat{\mathbf{x}}$ is

$$\begin{aligned}
& U(\hat{\mathbf{x}}, \sum_i [u_i(\hat{x}_1, \ldots, \hat{x}_{i-1}, \hat{x}_i, \hat{y}_{i+1}, \ldots, \hat{y}_N) - u_i(\hat{x}_1, \ldots, \hat{x}_{i-1}, o_i, \hat{y}_{i+1}, \ldots, \hat{y}_N)]) \\
= \; & U(\hat{\mathbf{x}}, \sum_i w_i [\Phi(\hat{x}_1, \ldots, \hat{x}_{i-1}, \hat{x}_i, \hat{y}_{i+1}, \ldots, \hat{y}_N) - \Phi(\hat{x}_1, \ldots, \hat{x}_{i-1}, o_i, \hat{y}_{i+1}, \ldots, \hat{y}_N)]).
\end{aligned}$$

She then chooses the optimal action profile $\mathbf{x}^* \in X$ to maximize the above payoff function. If $\hat{x}_j C \hat{x}_i$ for all $i, j \in N$ and all agents have the same weight of w, the payoff function is simplified to $U(\hat{\mathbf{x}}, w[\Phi(\hat{\mathbf{x}}) - \Phi(\mathbf{o})])$. Hence, we have the following corollary.

[16] Similar to footnote 15, we have $u_1(\hat{x}_1, \mathbf{x}_{-1}) - \hat{p}_1^* \geq u_1(o_1, \mathbf{x}_{-1})$ for all $\mathbf{x}_{-1} \in \prod_{i \neq 1}\{o_i, \hat{x}_i\}$. This is because $\hat{p}_1^* = u_1(\hat{x}_1, \hat{\mathbf{y}}_{-1}) - u_1(o_1, \hat{\mathbf{y}}_{-1})$ by (1.10) and $\hat{y}_i = o_i$ if $\hat{x}_i C \hat{x}_1$ and $\hat{y}_i = \hat{x}_i$ otherwise imply $u_1(\hat{x}_1, \mathbf{x}_{-1}) - u_1(o_1, \mathbf{x}_{-1}) \geq u_1(\hat{x}_1, \hat{\mathbf{y}}_{-1}) - u_1(o_1, \hat{\mathbf{y}}_{-1})$ for all $\mathbf{x}_{-1} \in \prod_{i \neq 1}\{o_i, \hat{x}_i\}$.

[17] Note that $\hat{\mathbf{x}}$ becomes the unique and strict Nash equilibrium if the principal charges each agent a price slightly lower than that of the $\mathbf{w}$-DC contracts. The enlargement of $F(\hat{\mathbf{x}})$ to its closure on p. 11 guarantees she can implement $\hat{\mathbf{x}}$ with the $\mathbf{w}$-DC contracts under any equilibrium selection criterion.

Corollary 2 *If $x_j C x_i$ and $w_i = w$ for all $i, j \in N$ and $\mathbf{x} \in X$, the principal's optimal action profile is*

$$\mathbf{x}^* \in \arg\max_{\mathbf{x} \in X} U(\mathbf{x}, w[\Phi(\mathbf{x}) - \Phi(\mathbf{o})]).$$

Recall from p. 8 that $x_j C x_i$ for all $i, j \in N$ and $\mathbf{x} \in X$ is much weaker than C1. Also, all agents having the same weight does not imply they are identical. Section 1.4 demonstrates how agents can be heterogeneous in many aspects while having the same weight.

1.3.1 Discussion

Interpretation of Theorem 1 A direct implication is that the principal should/would offer the $\mathbf{w}$-DC contracts as long as the underlying equilibrium selection criterion is quite pessimistic. In practice, the principal (or we, as researchers) may be uncertain about which equilibrium will arise when there are multiple equilibria. In this scenario, Theorem 1 implies that as long as she is not very optimistic, then she should/would, again, offer the $\mathbf{w}$-DC contracts. Note that even if the underlying equilibrium selection criterion is more optimistic (or not more pessimistic) than potential maximization, the $\mathbf{w}$-DC contracts can still be optimal for many games, just not for all games. Therefore, the use of the $\mathbf{w}$-DC contracts is indeed robust in many scenarios.

Agents' Ranking as Prioritization A high (low) ranking in the divide-and-conquer contracts means that the agent is (not) prioritized. In particular, the contract prices (1.10) are set in a way that the highest ranked agent 1 has a dominant strategy to accept the offer, whereas the lowest ranked agent N is indifferent between accepting and rejecting the offer in equilibrium. In the special case where all agents have the same weight, Proposition 1 shows that the principal can rank them in an arbitrary order. However, every agent wishes to be ranked higher because he would receive a lower price, or in other words, a more favorable contract.

Optimal Ranking When agents have different weights, Proposition 1 shows that the principal optimally ranks them in ascending order of weight w_i. To understand the rationale of this ranking, first consider the polar case where $\hat{x}_j C \hat{x}_i$ for all $i, j \in N$. This, together with (1.1), implies $\Phi(\hat{x}_i, \mathbf{x}_{-i}) - \Phi(o_i, \mathbf{x}_{-i})$ is increasing in $\mathbf{x}_{-i} \in \prod_{j \neq i}\{o_j, \hat{x}_j\}$ if we view o_j as 0 and $\hat{x}_j$ as 1. In other words, the potential function Φ captures the strategic complementarities among agents' target actions. Observe from (1.2) that agents with higher weights care more about Φ, i.e., they care more about the strategic complementarities. Therefore, by placing them at lower ranks in the (simple) divide-and-conquer contracts, the principal can extract more surplus. Now consider the other polar case where $\hat{x}_j S \hat{x}_i$ for all $i, j \in N$. Observe from (1.10) that each agent's price is $u_i(\hat{\mathbf{x}}) - u_i(o_i, \hat{\mathbf{x}}_{-i})$ no matter how we relabel the agents,[18] i.e., the ranking does not matter at all. In other words, the strategic substitutabilities among agents play no role in the ranking decision. The insights from these two polar cases carry over to the general case, and therefore the principal optimally ranks agents in increasing weight. A1 is based on agents' utilities $\mathbf{u}$, and therefore so is the weight vector $\mathbf{w}$.[19] Section 1.4 studies three examples of $\mathbf{u}$ and demonstrates how $\mathbf{w}$ differs in different contexts.

Implementation Requirements The previous analysis focuses on equilibrium selection criteria, which is one type of implementation requirements as explained on p. 10. I now extend Theorem 1 if we are also interested in other implementation requirements. Recall that an implementation requirement is fully characterized by $F \equiv (F(\mathbf{x}))_{\mathbf{x} \in \Delta(X)}$ where $F(\mathbf{x}) \subseteq P$. I say one implementation requirement F is *stronger* than the other F' if $F(\mathbf{x}) \subseteq F'(\mathbf{x})$ for all $\mathbf{x} \in \Delta(X)$. Note that "stronger" and "more pessimistic" are different

[18] In other words, every agent is indifferent between accepting and rejecting the offer in equilibrium when all target actions are strategic substitutes.

[19] Clearly, $\mathbf{w}$ is independent of the principal's payoff U. Shrinking agents' action sets X_i also has no impact on $\mathbf{w}$. In the context of capital raising, this implies that the optimal ranking to offer divide-and-conquer contracts is independent of agents' capital endowments; this is opposite to the main finding of Halac et al. (2020). The difference in results stems from the fact that their principal can only make payment contingent on a stochastic outcome but not on the chosen action.

concepts.[20] Let $\mathcal{F}$ denote the set of equilibrium selection criteria more pessimistic than potential maximization and $\bar{F}$ be an implementation requirement where $\bar{F}(\mathbf{x}) = \bigcup_{F\in\mathcal{F}} F(\mathbf{x})$. By construction, $\bar{F}$ is *weaker* than each $F \in \mathcal{F}$. Yet, the fact that the $\mathbf{w}$-DC contracts are optimal for all $F \in \mathcal{F}$ implies they remain optimal under $\bar{F}$. In addition, recall that the $\mathbf{w}$-DC contracts implement the target action profile as a dominance-solvable equilibrium, and therefore they remain feasible under the very strong requirement of *dominance-solvable implementation*. The above two findings imply the following corollary.

Corollary 3 *The* $\mathbf{w}$*-DC contracts are optimal for all games if the underlying implementation requirement is stronger than* $\bar{F}$ *and weaker than dominance-solvable implementation.*

Divide and Conquer Although the term "divide and conquer" is not formally defined in this literature, a coherent definition is "to implement a target action profile as a dominance-solvable equilibrium." Hence, the use of divide-and-conquer contracting schemes *per se* is not particularly interesting because it is tautological under the requirement of dominance-solvable implementation. An interesting result in the literature (see p. 4) is that sometimes divide-and-conquer contracting schemes remain optimal under the weaker requirement (but still stronger than every equilibrium selection criterion) of *unique implementation*, i.e., to implement a target action profile as the unique Nash equilibrium. Theorem 1 and Corollary 3 show that, for a large class of contracting models, a specific divide-and-conquer contracting scheme—the $\mathbf{w}$-DC contracts—is optimal for a large class of equilibrium selection criteria and implementation requirements. In other words, the use of the $\mathbf{w}$-DC contracts is a common phenomenon, which need not be justified by some strong implementation requirements.

Single Contract Lemma 2, Theorem 1, and Corollary 3 together imply that the principal can, without loss of optimality, offer one contract to each agent for a large class of

[20]In fact, before the enlargement (see p. 11), any equilibrium selection criterion F is not stronger than the other F' because $\{F(\mathbf{x})\}_{\mathbf{x}\in\Delta(X)}$ and $\{F'(\mathbf{x})\}_{\mathbf{x}\in\Delta(X)}$ are different partitions of P. This also implies that any implementation requirement strictly stronger than an equilibrium selection criterion rules out the use of certain menu profiles $\mathbf{p} \in P$.

equilibrium selection criteria and implementation requirements. This result is non-trivial because a major contribution of Segal (2003, Lemma 3) is to show that, under C1 and the requirement of unique implementation, the principal is generally better off offering multiple contracts to each agent. It turns out that just imposing A1–A3 already rules out this possibility.

1.4 Applications

This section applies the previous results to (i) networks, (ii) public goods, and (iii) impure public goods, and derives novel implications for each application. These applications together demonstrate how seemingly contradictory implications across applications are reconciled with the general theories developed in Section 1.3.

1.4.1 Networks

I now revisit the general example (1.3). Observe that it encompasses the two most popular forms of network games: the binary-action form if $X_i = \{0, 1\}$ and the linear-quadratic form if $b_i(x_i) = \alpha_i x_i - \beta_i x_i^2$. In the former, the principal's contracting scheme reduces to charging each agent a participation fee $p_i(1)$. Agents' utilities $\mathbf{u}$ clearly satisfy C1 (which implies A2 and A3); the following lemma shows that they also satisfy A1.

Lemma 4 *Agents' utilities constitute a weighted potential game with* $\mathbf{w} = \left(\frac{v_i}{\theta_i}\right)_i$ *and*

$$\Phi(\mathbf{x}) = \sum_i \frac{\theta_i b_i(x_i)}{v_i} + \frac{1}{2} \sum_i \sum_{j \in E_i} \theta_i \theta_j x_i x_j. \tag{1.12}$$

Therefore, all previous results apply to this example. Theorem 1 and Corollary 1 imply the following corollary.

Corollary 4 *Under any equilibrium selection criterion more pessimistic than potential maximization, the principal's optimal contracting scheme is to offer the* $\mathbf{w}$*-SDC contracts where* $\mathbf{w} = \left(\frac{v_i}{\theta_i}\right)_i$.

Agents' ranking in the $\mathbf{w}$-SDC contracts is based solely on their valuation-to-importance ratios $\frac{v_i}{\theta_i}$ but not X_i, b_i, or E_i. This generalizes the main finding of Sakovics and Steiner

(2012, Proposition 2), who study a binary-action complete network (i.e., $X_i = \{0, 1\}$ and $E_i = N\backslash\{i\}$) under *global-game selection* (which is equivalent to potential maximization as stated in footnote 5). They show that the optimal ranking depends on $\frac{v_i}{\theta_i}$ but not b_i. Corollary 4 shows that the ranking is also independent of agents' action sets X_i and sets of interacting agents E_i, and this result is robust to all equilibrium selection criteria more pessimistic than global-game selection. Contrary to the conventional wisdom that the principal should prioritize agents with important positions in the network (e.g., the center agent in a star network), the entire network structure actually plays no role in the ranking decision. Furthermore, as we will see more clearly in (1.13), an agent faces a higher, not lower, price if he interacts with more agents.

When all agents have the same weight, they can still differ in all other three dimensions (X_i, b_i, E_i). The literature on the economics of networks often assumes $v_i = \theta_i = 1$ for all i. This assumption implies $w_i = 1$ for all i, and therefore the principal's optimal action profile $\mathbf{x}^*$ is characterized by Corollary 2. Also, recall from p. 15 that the principal can rank agents arbitrarily in this case. This echoes the previous finding: the principal has no strict incentive to prioritize and offer more favorable contracts to agents with high centrality.

I now discuss further implications of this network model. For expositional convenience, assume $X_i = \{0, 1\}$ for all i, $\frac{v_1}{\theta_1} < \cdots < \frac{v_N}{\theta_N}$ (which implies $w_1 < \cdots < w_N$), and $\hat{\mathbf{x}} = \mathbf{x}^* = \mathbf{1}$. The $\mathbf{w}$-SDC contracts (1.11) are given by

$$p_i^*(1) = b_i(1) - b_i(0) + v_i \sum_{j \in E_i : w_j < w_i} \theta_j \quad \text{for all } i \in N. \tag{1.13}$$

Hence, agent i's equilibrium payoff is

$$u_i(\mathbf{1}) - p_i^*(1) = b_i(0) + v_i \sum_{j \in E_i : w_j > w_i} \theta_j.$$

Now consider a scenario in which agent i interacts with an additional agent $j \notin E_i$. Agent i is strictly better off if $w_j > w_i$: he pays the same price but derives additional interaction benefits. Conditional on $w_j > w_i$, he most prefers the additional agent with the highest importance θ_j. By contrast, agent i is just as well off if $w_j < w_i$: the principal raises his price by an amount equal to his additional interaction benefits. In any case, agent i does not mind interacting with more agents. Therefore, if the network is endogenously formed in stage 0, a natural formation process is that each agent unilaterally enables a few interactions. Under this process, agents with high weights $\frac{v_i}{\theta_i}$ and/or importance θ_i end up interacting with many agents in equilibrium. In other words, popular agents are those who value the network a lot and those with either high or low importance. If all agents have the same valuation and can only enable one interaction, an assortative line network is formed in which agent i chooses agent $i+1$ (agent N is indifferent between choosing any agent). To the best of my knowledge, these findings are novel to the literature on network formation.

1.4.2 Public Goods

I now formalize the example described on p. 9. Recall that building a public facility involves many incomparable actions. Therefore, I keep each agent's action set X_i as the most general form, i.e., it can be any compact set. Agent i's utility takes the following form:

$$u_i(\mathbf{x}) = b_i(x_i) + v_i g(\mathbf{x}), \tag{1.14}$$

where $b_i : X_i \to \mathbb{R}$ measures his stand-alone benefit/cost, $v_i \in \mathbb{R}_{++}$ measures his valuation of the public good, and $g : X \to \mathbb{R}$ measures the size of the public good. Agents can differ in four dimensions: (X_i, b_i, v_i) and how each agent's actions x_i affect the size of the public good g. The function g can be very general (but not arbitrary as I will explain shortly), and it captures the nature (in particular, the importance) of each agent's contribution to the public good. For example, if $o_i = 0 \in X_i \subseteq \mathbb{R}_+$ and $g(\mathbf{x}) = (\sum_j \theta_j x_j)^2$, then $\theta_j \in \mathbb{R}_{++}$ measures the relative importance of agent j's actions as in the network model (1.3). The following lemma shows that agents' utilities $\mathbf{u}$ satisfy A1.

Lemma 5 *Agents' utilities constitute a weighted potential game with* $\mathbf{w} = (v_i)_i$ *and*

$$\Phi(\mathbf{x}) = \sum_i \frac{b_i(x_i)}{v_i} + g(\mathbf{x}). \tag{1.15}$$

To state the condition for $\mathbf{u}$ to satisfy A2 and A3, I first define the modified binary relations C_g and S_g as follows.

Definition 3 The expression $x_j C_g x_i$ ($x_j S_g x_i$) stands for

$$g(x_i, x_j, \mathbf{x}_{-ij}) - g(o_i, x_j, \mathbf{x}_{-ij}) \geq (\leq)\ g(x_i, o_j, \mathbf{x}_{-ij}) - g(o_i, o_j, \mathbf{x}_{-ij}) \quad \forall \mathbf{x}_{-ij} \in X_{-ij}.$$

We can easily verify that $x_j C x_i$ ($x_j S x_i$) if and only if $x_j C_g x_i$ ($x_j S_g x_i$). Therefore, A2 and A3 hold if and only if they remain true when C and S are replaced by C_g and S_g respectively. Suppose they indeed remain true. Then all results in Section 1.3 apply to this model; Theorem 1 implies the following corollary.

Corollary 5 *Under any equilibrium selection criterion more pessimistic than potential maximization, the principal's optimal contracting scheme is to offer the* $\mathbf{w}$*-DC contracts where* $\mathbf{w} = (v_i)_i$*.*

Agents' ranking in the $\mathbf{w}$-DC contracts is based solely on their valuations of the public good v_i but not X_i, b_i, or g. In contrast to the network model where the optimal ranking depends crucially on agents' importance θ_i by Corollary 4, the ranking is now independent of their importance to the public good (as captured by g). The reason for these opposing results is that the public good is *non-excludable* whereas the "network good" is *excludable*, in which an agent derives zero interaction benefit whenever he rejects the offer. In other words, the principal's optimal contracting scheme depends critically on the excludability of externalities. Section 1.4.3 formalizes the above arguments. Recall from p. 18 that the main finding of Sakovics and Steiner (2012) is a special case of Corollary 4 and, therefore, does not apply to public goods or, more generally, non-excludable externalities. This urges caution in applying their proposed contracting scheme to their leading applications: economic development and financial fragility; both are largely public goods/bads in nature or at least partially non-excludable. Section 1.4.3 proposes a refined contracting scheme.

1.4.3 Impure Public Goods

Consider a hybrid of the previous network and public good models. Agent i's action set X_i is a compact subset of $\mathbb{R}_+$ with $o_i = 0 \in X_i$. Agent i's utility takes the following form:

$$u_i(\mathbf{x}) = b_i(x_i) + \underbrace{\delta v_i x_i \sum_j \theta_j x_j}_{\text{excludable externalities}} + \underbrace{\frac{(1-\delta)v_i}{2}(\sum_j \theta_j x_j)^2}_{\text{public-good externalities}} + \underbrace{(1-\delta)\xi_i(\mathbf{x}_{-i})}_{\text{pure externalities}},$$

where $\delta \in [0,1]$ measures the degree of excludability, $\xi_i : X_{-i} \to \mathbb{R}$ measures the pure externalities generated by others' actions, and $b_i : X_i \to \mathbb{R}$, $v_i \in \mathbb{R}_{++}$, and $\theta_j \in \mathbb{R}_{++}$ are interpreted the same way as before. Both public-good and pure externalities are non-excludable, but the latter play no strategic role. In fact, adding arbitrary pure externalities to agents' utilities in the previous models makes them more general and has no impact on subsequent results. If $\delta = 1$, this model reduces to the network model (1.3) with a complete network. If $\delta = 0$, it reduces to the public good model (1.14) with $o_i = 0 \in X_i \subseteq \mathbb{R}_+$ and $g(\mathbf{x}) = \frac{1}{2}(\sum_j \theta_j x_j)^2$. Agents' utilities clearly satisfy C1; they also satisfy A1.[21]

Lemma 6 *Agents' utilities constitute a weighted potential game with* $\mathbf{w} = \left(\delta \frac{v_i}{\theta_i} + (1-\delta)v_i\right)_i$ *and*

$$\Phi(\mathbf{x}) = \sum_i \frac{b_i(x_i) + \frac{1}{2}\delta v_i \theta_i x_i^2}{w_i} + \frac{1}{2}(\sum_i \theta_i x_i)^2. \tag{1.16}$$

Therefore, Theorem 1 and Corollary 1 imply the following corollary.

Corollary 6 *Under any equilibrium selection criterion more pessimistic than potential maximization, the principal's optimal contracting scheme is to offer the* $\mathbf{w}$*-SDC contracts where* $\mathbf{w} = \left(\delta \frac{v_i}{\theta_i} + (1-\delta)v_i\right)_i$.

In this model, the optimal ranking is determined by the weighted average of each agent's valuation-to-importance ratio $\frac{v_i}{\theta_i}$ (which is the only determinant in the network model) and his valuation v_i (which is the only determinant in the public good model), and

[21] The fact that this model is (a special case of) a convex combination of the previous two models does not imply it satisfies A1 because a convex combination of two weighted potential games need not be a weighted potential game. Furthermore, even if the combination turns out to be a weighted potential game, the corresponding weights need not be a convex combination of the previous weights. Therefore, what the following lemma shows are specific to this model rather than some universal facts.

the relative weight depends on the degree of excludability δ. In other words, the principal always prioritizes agents with low valuations, whereas she also prioritizes those with high importance if and only if the externalities are sufficiently excludable. As mentioned, economic development and financial fragility are at least partially non-excludable, in that almost everyone benefits from a strong economy and suffers from a financial crisis. Hence, Corollary 6 provides theoretical guidance on sectoral industrial policies for the former and financial policies for the latter. Note that agents' action sets X_i play no role in the optimal ranking for this and all previous models. In other words, the principal need not prioritize "large" agents who can take large actions $x_i \in X_i$ and affect everyone significantly. This suggests that the well-known "too-big-to-fail" doctrine of bailing out large financial institutions (say, ranking agents in decreasing $\max X_i$ in the divide-and-conquer contracts) may be suboptimal for preventing financial crises.

1.5 Conclusion

This paper addresses the question of what contracts the principal should or would offer when there are multiple agents. For a large class of contracting models, I show that the $\mathbf{w}$-DC contracts are optimal for a large class of equilibrium selection criteria and implementation requirements. This result provides robust predictions and policy guidance for a wide variety of applications especially when we, as researchers, do not know the underlying equilibrium selection criterion or what implementation requirement the principal wants to meet. Finally, the general framework, newly developed tools, and the utilization of potential game theory promise to open a wide range of new research opportunities in multi-agent contracting.

Chapter 2

Divide and Conquer in Two-Sided Markets: A Potential-Game Approach

2.1 Introduction

Two groups of agents often interact via platforms: men and women meet in a nightclub, buyers and sellers trade on a marketplace, consumers and merchants transact through a payment card, and so on. These markets are known as two-sided markets. Typically, positive cross-side network effects are present in these markets, creating strategic complementarities among agents. For example, a man (woman) wants to join a heterosexual nightclub only if some women (men) join the nightclub. Therefore, to attract men, the nightclub needs to attract many women, but to attract women, the nightclub needs to attract many men. This issue is known as the classic "chicken-and-egg" problem (Caillaud and Jullien 2003), one of the most difficult challenges for many two-sided platforms and a methodological challenge for researchers on two-sided markets.

Formally, in a typical two-sided market model where platforms set prices in stage 1 and all agents simultaneously make their participation decisions in stage 2, agents often engage in a coordination game with multiple Nash equilibria. For example, when there is a monopoly platform and agents from the same side are identical, there can be two equilibria in stage 2: (i) all agents join the platform and (ii) no one joins the platform. When there are competing platforms, most (if not all) agents will coordinate on one of the platforms in equilibrium if network effects are sufficiently strong; but which one will they coordinate on? In other words, how should we deal with this multiple equilibria issue?

As I will elaborate in the Related Literature, researchers on two-sided markets impose various selection criteria to get rid of multiple equilibria. However, each selection criterion only works for some models but not others. Moreover, different selection criteria often lead to conflicting predictions and implications. Therefore, there is a methodological challenge of selecting a suitable equilibrium. In response to this challenge, this paper proposes using a refinement of Nash equilibrium, called *potential maximization*, to resolve the multiplicity of Nash equilibria in two-sided markets. As I will explain, this refinement is justified by many solid microfoundations in the game theory literature, widely supported by experimental evidence, well describes agent behavior in two-sided markets, applicable to many two-sided market models, yields realistic predictions, and very tractable.

To illustrate the aforementioned challenge and proposed solution, consider the following example where a woman (row player) and a man (column player) decide whether to join a nightclub:

	Join	Not join
Join	$v_1 - p_1, v_2 - p_2$	$-p_1, 0$
Not join	$0, -p_2$	$0, 0$

(2.1)

For a wide range of fees $(p_1, p_2) \in [0, v_1] \times [0, v_2]$ set by the nightclub, both (Join, Join) and (Not join, Not join) are equilibria. A frequently used selection criterion, *Pareto dominance*, selects the former whenever there are multiple equilibria, leading to the prediction that the nightclub optimally sets $p_1^* = v_1$ and $p_2^* = v_2$. It also predicts that both agents join the nightclub and derive zero payoff in equilibrium, even though each risks a negative payoff of $-v_i$ if the other does not show up and can guarantee him/herself a zero payoff by not joining the nightclub.

I now turn to potential maximization, which coincides with the well-known *risk dominance* criterion (Harsanyi and Selten 1988) in 2-by-2 games. By definition,[1] (Join, Join) is the risk-dominant equilibrium if and only if $(v_1 - p_1)(v_2 - p_2) \geq p_1 p_2$, which is simplified to $p_1/v_1 + p_2/v_2 \leq 1$. Intuitively, risk dominance predicts that both agents will join the

[1] Generally, (J, J) risk dominates (N, N) if $[u_1(J, J) - u_1(N, J)][u_2(J, J) - u_2(J, N)] \geq [u_1(N, N) - u_1(J, N)][u_2(N, N) - u_2(N, J)]$.

nightclub only when it is relatively safe to do so. Hence, the nightclub maximizes $p_1 + p_2$ subject to $p_1/v_1 + p_2/v_2 \leq 1$. If the man enjoys the nightclub more than the woman (i.e., $v_2 > v_1$) even by just a little, the nightclub will optimally set $p_1^* = 0$ and $p_2^* = v_2$: this is the "ladies' night" phenomenon. More generally, the above pricing strategy that subsidizes one side and charges the other is called "divide and conquer" in the literature.

Besides nightclubs, many other two-sided platforms also divide and conquer in practice: shoppers visit shopping malls for free while retailers pay the rent; consumers are paid to use credit cards while merchants pay for the service; open access journals are freely available to readers while authors pay submission and publication fees (the latter is US$3,100 for the open access option of *The RAND Journal of Economics*); buyers receive offers from daily deal sites (e.g., Groupon, LivingSocial) at no charge while sellers typically split the revenue 50/50 with the sites (Dholakia 2011), and the list goes on.

All the above examples involve more than two agents, making risk dominance inapplicable. Nevertheless, a generalization of risk dominance, potential maximization, is applicable to many multi-agent models. To introduce this selection criterion in the simplest way, Section 2.2 extends the illustrative example (2.1) with multiple identical agents on each side; it is also the identical-agent version of Armstrong's (2006, Section 3) monopoly model. Similar to (2.1), under potential maximization, the platform has to ensure sufficiently low miscoordination cost by pricing low enough in stage 1 so that all agents will join the platform in stage 2. Again, the platform's optimal pricing strategy is to divide and conquer. The sole determinant of which side to divide (i.e., subsidize) or conquer (i.e., monetize) is the relative size of cross-side network effects (which corresponds to whether v_1 or v_2 is larger in (2.1)), i.e., independent of the number of agents on each side and the costs of serving the agents. This divide-and-conquer strategy implies that the optimal design of the profit-maximizing platform is to favor the "conquer side" only, which is socially suboptimal.

Section 2.3 studies platform competition under potential maximization and derives further insights into two-sided markets. The model in Section 2.2 is naturally extended with two potentially asymmetric platforms. Under potential maximization, the platforms' price-setting stage is analogous to standard Bertrand competition. In equilibrium, the entire market tips to an endogenously determined platform. Again, this dominant platform always divides and conquers. Which side to divide/conquer now depends on the relative size of average cross-side network effects across the competing platforms instead of its own cross-side network effects of the two sides. The optimal design of platforms now tends to favor both sides (the conquer side) when the platforms are very (not) competitive.

Section 2.4 further justifies the use of potential maximization in two-sided markets and compares it with selection criteria commonly used in this literature. Section 2.5 studies various extensions to show the generality of potential maximization and derive further insights into two-sided markets. In particular, potential maximization is applicable to models with (i) alternative pricing instruments such as transaction fees and two-part tariffs, (ii) heterogeneous agents in terms of their stand-alone and interaction benefits from joining a platform as well as their importance to agents on the other side, (iii) more than two platforms, (iv) price discrimination, (v) same-side network effects, (vi) more than two stages, and (vii) an infinite number of agents. Section 2.6 concludes. The rest of this section reviews the related literature.

2.1.1 Related Literature

This paper contributes to the literature on two-sided markets pioneered by Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), and Armstrong (2006). The latter three study the case where agents are sufficiently heterogeneous to guarantee a unique equilibrium in the model. Their common finding is that a platform will divide and conquer if and only if the elasticities of demand or cross-side network effects of the two sides differ a lot. By contrast, agents from the same side are identical in Caillaud and Jullien's (2003) model, and therefore multiple equilibria arise in their duopoly model. To single out an equilibrium, they impose *monotonicity*, i.e., to assume the number of agents on each side joining a platform decreases with the platform's prices. When they study two modified models

(Sections 3 and 5), they impose both monotonicity and *focality*,[2] i.e., to assume all agents always coordinate on a pre-specified platform whenever multiple equilibria exist. Under their selection criteria, the competing platforms may divide and conquer (see also Jullien 2011). In contrast to all these seminal papers, Section 2.2 of this paper derives the divide-and-conquer strategy in an identical-agent monopoly model under potential maximization. In other words, the use of this strategy need not rely on heterogeneous agents or platform competition.

Other popular selection criteria in this literature are *Pareto dominance*, i.e., to select the Pareto-dominant equilibrium; *coalition-proofness* (e.g., Ambrus and Argenziano 2009; Karle et al. 2020), i.e., to allow joint deviations by any subset of agents; and *insulating tariffs* (e.g., Weyl 2010; White and Weyl 2016), i.e., to allow platforms' prices contingent on participation decisions of agents on all platforms. To the best of my knowledge, this paper is the first to introduce potential maximization into two-sided markets.

Potential maximization refines Nash equilibrium in *potential games*, a concept introduced by Rosenthal (1973) and formalized by Monderer and Shapley (1996). Section 2.2.3 explains both concepts in detail. Many equilibrium selection criteria in the game theory literature coincide with potential maximization if a game is a supermodular weighted potential game.[3] For example, Frankel et al. (2003) prove that the unique equilibrium under *global-game selection* (Carlsson and van Damme 1993; Morris and Shin 2003) is the *potential maximizer*, the equilibrium pinned down by potential maximization. Under *perfect foresight dynamics* (Matsui and Matsuyama 1995), the potential maximizer is the unique absorbing and globally accessible state (Hofbauer and Sorger 1999, 2002; Oyama et al. 2008). Under *log-linear dynamics*, the potential maximizer is the unique stochastically stable state (Blume 1993; Young 1998; Okada and Tercieux 2012; see also Myatt and Wallace

[2] This selection criterion is also called good/bad or favorable/unfavorable expectations (e.g., Caillaud and Jullien 2001; Hagiu 2006), optimistic/pessimistic beliefs (e.g., Caillaud and Jullien 2003), or incumbency advantage (e.g., Biglaiser et al. 2019).

[3] This implies that we can apply other selection criteria to a potential game and obtain the same prediction. Nevertheless, potential maximization is the most tractable approach because there is no need to introduce auxiliary incomplete information or dynamic elements, whereas others do.

2009). Ui (2001) and Morris and Ui (2005) prove that the potential maximizer is robust to incomplete information in the sense of Kajii and Morris (1997). All the selection criteria above coincide with risk dominance in 2-by-2 games. In addition to game-theoretic justifications, potential maximization is supported by ample experimental evidence (Van Huyck et al. 1990; Goeree and Holt 2005; Chen and Chen 2011).[4] I provide further justifications in Section 2.4.

Although potential maximization applies only to potential games, many two-sided market models are potential games (precisely, every subgame of these two-stage models is a potential game). In particular, Section 2.5.2 shows that the main models of all four aforementioned seminal papers are weighted potential games. Therefore, we can resolve the multiplicity of equilibria in all these models using the same approach: potential maximization.

2.2 Monopoly Platform

2.2.1 Model

This baseline model is a special case of Armstrong's (2006, Section 3) model in which agents from the same side are identical. A platform serves two sides of agents, indexed by 1 and 2, and there are N_1 side-1 agents and N_2 side-2 agents. The game has two stages. In stage 1, the platform sets subscription fees $(p_1, p_2) \in \mathbb{R}^2$ to the two sides. In stage 2, all agents observe p_1 and p_2 and simultaneously decide whether to join the platform. If the platform attracts n_1 side-1 agents and n_2 side-2 agents, the payoff of a side-i agent from joining the platform is

$$u_i(n_j, p_i) = v_i n_j - p_i, \quad (i, j = 1, 2;\ i \neq j) \tag{2.2}$$

[4] Anderson et al. (2001) introduce the notion of stochastic potential by adding some noise to the standard potential (the former converges to the latter as the noise goes to zero). Goeree and Holt (2005) and Chen and Chen (2011) find that subjects often end up at the maximizer of the stochastic potential.

where $v_i \in \mathbb{R}_{++}$ is the benefit of a side-i participant from interacting with each side-j participant. If an agent does not join the platform, his payoff is zero. The platform's payoff is equal to its profit:

$$\pi(n_1, n_2, p_1, p_2) = (p_1 - c_1)\, n_1 + (p_2 - c_2)\, n_2,$$

where $c_i \in \mathbb{R}_+$ is the (sufficiently low) cost of serving each side-i participant. The illustrative example (2.1) is the case where $N_1 = N_2 = 1$.

2.2.2 Pareto Dominance

I examine the subgame-perfect equilibria of this game. Multiple equilibria often arise in stage 2 due to cross-side network effects. In particular, there are two (generically strict)[5] equilibria when $(p_1, p_2) \in [0, v_1 N_2] \times [0, v_2 N_1]$: (i) all agents join the platform and (ii) no one joins the platform. Clearly, the former Pareto dominates the latter for all agents. As a benchmark, I characterize the equilibrium outcome under *Pareto dominance*, i.e., to select the full-participation equilibrium whenever there are multiple equilibria.

Proposition 1 *Under Pareto dominance, the platform sets $p_1^* = v_1 N_2$ and $p_2^* = v_2 N_1$. All agents join the platform with zero surplus, and the platform's equilibrium profit is $\pi^* = (v_1 + v_2) N_1 N_2 - c_1 N_1 - c_2 N_2$.*

For this baseline model, it is easy to see that the equilibrium outcomes under focality (with the platform being "focal"), coalition-proofness, and insulating tariffs are all given by Proposition 1.[6] Therefore, this proposition in fact represents the prediction under a typical selection criterion.

[5] Throughout this paper, there are some non-strict equilibria such as mixed-strategy equilibria. Nevertheless, it is without loss of generality to ignore them because none of the popular selection criteria or potential maximization (see footnote 7) will select them.

[6] Armstrong (2006, p. 672) allows the platform to directly choose agents' utility levels instead of setting prices; this also leads to the same prediction as in Proposition 1.

2.2.3 Potential Maximization

Now, I first introduce potential games and potential maximization, and then I will analyze the model under potential maximization. First, fixing an arbitrary subgame with $\mathbf{p} \equiv (p_1, p_2) \in \mathbb{R}^2$ set by the platform in stage 1, consider the following function:

$$\Phi_{\mathbf{p}}(n_1, n_2) = n_1 n_2 - \frac{p_1}{v_1} n_1 - \frac{p_2}{v_2} n_2. \tag{2.3}$$

Observe from (2.2) and (2.3) that for $i, j = 1, 2$ ($i \neq j$),

$$u_i(n_j, p_i) - 0 = v_i[\Phi_{\mathbf{p}}(n_i, n_j) - \Phi_{\mathbf{p}}(n_i - 1, n_j)]. \tag{2.4}$$

In other words, the function $\Phi_{\mathbf{p}}$ is constructed in a way that whenever an agent deviates, the change in $\Phi_{\mathbf{p}}$ is proportional to the change in the agent's payoff (the number of side-i participants decreases by one if a side-i agent switches from joining to not joining the platform). Thus, all agents' strategic considerations, which concern only unilateral deviations, are summarized by this single function $\Phi_{\mathbf{p}}$. A game is called a (*weighted*) *potential game* if it is possible to construct such a function $\Phi_{\mathbf{p}}$, which is also called a *potential function*. And I have just shown that every subgame in stage 2 is a weighted potential game. Note that the potential function (2.3) of this model depends only on the numbers of side-1 and side-2 participants because agents from the same side are identical. When Section 2.5.2 extends the model to allow for heterogeneous agents, the potential function would have to depend on agents' action profile. There I introduce more notation and give the general definition of a weighted potential game.

The potential function $\Phi_{\mathbf{p}}$ is a real-valued function, which clearly has a (generically) unique maximizer. Furthermore, this *potential maximizer* is a pure-strategy[7] Nash equilibrium: if someone deviates from the maximizer, the potential will decrease, and, by (2.4), the deviator will have a lower payoff. Hence, if there is a unique equilibrium, the potential

[7] The potential of a mixed-strategy equilibrium is a convex combination of the potentials defined on the set of pure-strategy action profiles. Therefore, generically a mixed-strategy equilibrium is not a potential maximizer.

maximizer is the unique equilibrium; if there are multiple equilibria, the potential maximizer is one with the highest potential. Moreover, as explained in the Related Literature, many theoretical and experimental studies show that agents often end up at the potential maximizer. Therefore, this maximizer refines Nash equilibrium in weighted potential games, and this refinement is called *potential maximization.* Section 2.4 further justifies this refinement.

Recall that every subgame of the current model is a weighted potential game. By applying potential maximization to each subgame, we resolve the multiplicity of Nash equilibria. The potential maximizer of each subgame is given as follows.

Lemma 1 *When $p_1, p_2 \geq 0$, the potential maximizer is all agents joining the platform if $\frac{p_1}{v_1 N_2} + \frac{p_2}{v_2 N_1} \leq 1$ and no one joining the platform otherwise.*[8]

Proof. The only non-trivial case is when $(p_1, p_2) \in [0, v_1 N_2] \times [0, v_2 N_1]$, in which there are two equilibria. By (2.3), their respective potentials are

$$\Phi_{\mathbf{p}}(N_1, N_2) = N_1 N_2 - \frac{p_1}{v_1} N_1 - \frac{p_2}{v_2} N_2, \quad \Phi_{\mathbf{p}}(0, 0) = 0,$$

and the potential maximizer is the one with the higher potential. ■

As shown in Lemma 1, under potential maximization, the platform has to leave enough surplus to agents by setting sufficiently low prices (p_1, p_2) in stage 1 so that all agents will join the platform in stage 2. Hence, its profit maximization problem is

$$\max_{p_1, p_2 \geq 0} (p_1 - c_1) N_1 + (p_2 - c_2) N_2 \quad \text{s.t.} \quad \frac{p_1}{v_1 N_2} + \frac{p_2}{v_2 N_1} \leq 1.$$

The solution to this problem is given as follows.

Proposition 2 *Suppose $v_1 < v_2$. Under potential maximization, the platform sets $p_1^* = 0$ and $p_2^* = v_2 N_1$. All agents join the platform, and the platform's equilibrium profit is $\pi^* = v_2 N_1 N_2 - c_1 N_1 - c_2 N_2$.*

[8] Both equilibria are potential maximizers when $\frac{p_1}{v_1 N_2} + \frac{p_2}{v_2 N_1} = 1$. Nevertheless, the platform can lower p_1 or p_2 a bit so that the full-participation equilibrium is the unique potential maximizer. Therefore, for expositional convenience, I assume that the full-participation equilibrium is selected when $\frac{p_1}{v_1 N_2} + \frac{p_2}{v_2 N_1} = 1$.

2.2.4 Discussion

I now discuss and compare the predictions under Pareto dominance and potential maximization as summarized by Propositions 1 and 2 respectively. In both cases, the platform charges side-2 agents the same maximum price and fully extracts their surplus. By contrast, under potential maximization, the platform provides free access to side-1 agents and leaves them a lot of surplus. Therefore, its equilibrium profit is much lower than that under Pareto dominance. There are three key implications in this model.

Divide and Conquer Under potential maximization, the platform's *divide-and-conquer* strategy that subsidizes one side (hereafter the *divide side*) and monetizes the other (hereafter the *conquer side*) is ubiquitous because $v_1 \neq v_2$ in general. Indeed, this pricing strategy is widely observed in reality (see p. 26 for five examples). To derive this strategy in the current model without using potential maximization, one would need to rely on *Pareto-dominated selection*, i.e., to select the Pareto-dominated equilibrium whenever multiple equilibria exist. However, this selection criterion is often regarded as even "less plausible" than Pareto dominance (as elaborated in footnote 17). Surprisingly, the prediction under Pareto-dominated selection coincides with that under potential maximization in this baseline model.[9] Yet, this equivalence is a knife-edge result: it no longer holds when I extend the model in subsequent sections.

Divide/Conquer Side The only determinant of which side to divide or conquer is the relative size of per-interaction benefits v_1 and v_2. In other words, the divide/conquer side is independent of the number of agents N_1 and N_2 on each side, suggesting that the platform need not monetize the side with more agents. For example, shopping malls have more shoppers than retailers, but only the latter are charged. This is because when there are more, say, side-1 agents, the platform can extract more surplus from side 1. Meanwhile,

[9] Under Pareto-dominated selection, the platform has to guarantee participation from one side by providing free access to that side. It then can charge the other side the maximum price. Clearly, the choice of the divide/conquer side is the same as that under potential maximization.

having more side-1 agents increases the benefits to side-2 agents, and thus the platform can also extract more surplus from side 2. These two effects cancel out in this model, and therefore the divide/conquer side does not depend on N_1 and N_2.

Similarly, the divide/conquer side is independent of the costs of serving the participants c_1 and c_2. For example, for open access journals, the marginal cost of an additional reader is zero whereas reviewing a paper is costly; yet, these journals only charge authors. This is because in the model, all agents coordinate on either joining or not joining the platform in equilibrium. Therefore, the total cost $c_1N_1 + c_2N_2$ incurred by the platform is equivalent to a fixed cost, which is irrelevant to the decision on the divide/conquer side.

Optimal Design Oftentimes, agents' per-interaction benefits v_1 and v_2 are not exogenous, but rather the platform's endogenous choice. For example, shopping malls are often designed to maximize shoppers' travel distances by locating anchor stores far from each other and placing escalators at opposite ends; this benefits retailers but harms shoppers. The following discussion investigates the comparative statics of v_1 and v_2.

Under Pareto dominance, the optimal design of the platform is to favor both sides, i.e., to increase both v_1 and v_2. This is not true under potential maximization: the platform's equilibrium profit π^* is independent of v_1 as long as $v_1 < v_2$. Therefore, it only has the incentive to increase v_2, i.e., the optimal design of the platform (e.g., shopping mall) is to favor only the conquer side (e.g., retailers). In addition, under Pareto dominance, social surplus is equal to the platform's equilibrium profit. This implies that the optimal design of the platform also maximizes social surplus. By contrast, under potential maximization, the optimal design of the platform is socially suboptimal because it has no incentive to increase side-1 agents' surplus by increasing v_1.

This section highlights how different selection criteria can lead to completely different predictions and implications: this is the methodological challenge in two-sided markets. Nevertheless, potential maximization, a refinement of Nash equilibrium justified by many theoretical and experimental studies, yields more realistic predictions in this identical-agent monopoly model. Section 2.4.2 further substantiates how potential maximization better describes agent behavior in two-sided markets than Pareto dominance does. In fact, the above predictions already capture many distinctive features of two-sided markets. In other words, these features need not rely on heterogeneous agents or platform competition; rather, they rely on a suitable selection criterion.

2.3 Platform Competition

This section studies platform competition under potential maximization and derives further insights into two-sided markets. The previous model is extended to a duopoly model, which is closely related to Armstrong's (2006, Section 4) and Caillaud and Jullien's (2003, Section 5) models.[10] Multiple equilibria naturally arise in their models, but they do not attempt to pin down an equilibrium.[11] By contrast, I analyze the model under potential maximization and derive a unique prediction. For most platform competition models including the current model, *focality* is the only popular selection criterion that can single out a unique outcome (which differs from that of this section). Section 2.4.3 compares and contrasts potential maximization with focality.

2.3.1 Model

There are now two competing platforms, indexed by A and B. In stage 1, both platforms simultaneously set prices p_1^m and p_2^m for the two sides ($m = A, B$). In stage 2, all agents simultaneously decide which platform to join. Let n_i now denote the number of side-i agents joining platform A. The respective payoffs of a side-i agent from joining A and B are

$$u_i^A(n_j, p_i^A) = v_i^A n_j - p_i^A, \quad u_i^B(n_j, p_i^B) = v_i^B(N_j - n_j) - p_i^B. \tag{2.5}$$

[10] A major difference is that my model allows for asymmetric platforms whereas theirs do not. For symmetric platforms, mine is a special case of Armstrong's model with zero transport cost for agents (i.e., $t_1 = t_2 = 0$ in his model).

[11] In fact, Armstrong (2006) does not analyze the special case of $t_1 = t_2 = 0$ (see footnote 10); he only analyzes the case where t_1 and t_2 are sufficiently large so that there is a unique market-sharing equilibrium.

Observe that side-i agents may derive different per-interaction benefits $v_i^m \in \mathbb{R}_{++}$ at different platforms. For simplicity, I assume away the costs of serving the participants.[12] Hence, the platforms' profits are

$$\pi^A(n_1, n_2, p_1^A, p_2^A) = p_1^A n_1 + p_2^A n_2, \quad \pi^B(n_1, n_2, p_1^B, p_2^B) = p_1^B(N_1 - n_1) + p_2^B(N_2 - n_2).$$

Following Armstrong and Wright (2007), I assume the subscription fees p_1^m and p_2^m set by the platforms are non-negative.[13] They argue (p. 356) that this is a reasonable restriction for subscription models because strictly subsidizing participants will create obvious adverse selection and moral hazard problems. Armstrong (2006, footnote 5) makes a similar argument. Note that this model reduces to the baseline model if $v_1^B = v_2^B = 0$ and platform B is forced to set $p_1^B = p_2^B = 0$.

2.3.2 Analysis

Similar to the previous model, multiple equilibria often arise in stage 2. Let $\tilde{p}_1 \equiv p_1^A - p_1^B$ and $\tilde{p}_2 \equiv p_2^A - p_2^B$ denote the respective price differences between the two platforms. There are two equilibria when $(\tilde{p}_1, \tilde{p}_2) \in [-v_1^B N_2, v_1^A N_2] \times [-v_2^B N_1, v_2^A N_1]$:[14] (i) all agents join A and (ii) all agents join B. Neither Pareto dominance nor Pareto-dominated selection is applicable to this model because coordinating on one of the platforms need not Pareto dominate the other. By contrast, potential maximization remains applicable. Analogous to Section 2.2.3, we can verify that every subgame with $\mathbf{p} \equiv (p_1^A, p_2^A, p_1^B, p_2^B) \in \mathbb{R}^4$ set by platforms in stage 1 is a weighted potential game with the potential function

$$\Phi_{\mathbf{p}}(n_1, n_2) = n_1 n_2 - \frac{v_1^B N_2 + \tilde{p}_1}{v_1^A + v_1^B} n_1 - \frac{v_2^B N_1 + \tilde{p}_2}{v_2^A + v_2^B} n_2. \tag{2.6}$$

[12] With positive and asymmetric costs c_i^m across platforms, the identity of the dominant platform (as characterized by Proposition 3) would also depend on their cost-effectiveness.

[13] This restriction merely ensures that platforms' price competition in stage 1 is well-behaved. Without this restriction, every subgame in stage 2 remains a weighted potential game, and therefore potential maximization can always resolve the multiplicity of equilibria.

[14] The equilibrium is unique if prices are outside this range. As we will see, platforms' equilibrium prices (as characterized by Proposition 3) indeed fall into this range.

Precisely, we can show that for $i, j = 1, 2$ ($i \neq j$),

$$u_i^A(n_j, p_i^A) - u_i^B(n_j, p_i^B) = (v_i^A + v_i^B)[\Phi_{\mathbf{p}}(n_i, n_j) - \Phi_{\mathbf{p}}(n_i - 1, n_j)].$$

In words, fixing others' participation decisions, the payoff difference between joining A and B for each agent is proportional to the corresponding difference in $\Phi_{\mathbf{p}}$. Similar to the baseline model, the issue of multiple equilibria can be resolved by selecting the potential maximizer for each subgame, which is given as follows.

Lemma 2 *When $(\tilde{p}_1, \tilde{p}_2) \in [-v_1^B N_2, v_1^A N_2] \times [-v_2^B N_1, v_2^A N_1]$, the potential maximizer is all agents joining platform A if*

$$v_1^A v_2^A - \left(\frac{v_2^A + v_2^B}{N_2} p_1^A + \frac{v_1^A + v_1^B}{N_1} p_2^A\right) \geq v_1^B v_2^B - \left(\frac{v_2^A + v_2^B}{N_2} p_1^B + \frac{v_1^A + v_1^B}{N_1} p_2^B\right),$$

and all agents joining platform B otherwise.

Proof. By (2.6), the respective potentials of the two equilibria are

$$\Phi_{\mathbf{p}}(N_1, N_2) = \frac{v_1^A v_2^A - v_1^B v_2^B}{(v_1^A + v_1^B)(v_2^A + v_2^B)} N_1 N_2 - \frac{\tilde{p}_1}{v_1^A + v_1^B} N_1 - \frac{\tilde{p}_2}{v_2^A + v_2^B} N_2, \quad \Phi_{\mathbf{p}}(0, 0) = 0,$$

and the potential maximizer is the one with the higher potential. ■

As shown in Lemma 2, under potential maximization, we can view all agents as a "representative agent" who either joins A or B: the "value" of platform m is $v_1^m v_2^m$, its "price" is $\frac{v_2^A + v_2^B}{N_2} p_1^m + \frac{v_1^A + v_1^B}{N_1} p_2^m$, and the representative agent joins the platform with the higher "net value." Thus, stage 1 is analogous to standard Bertrand competition. Generically and w.l.o.g., assume A has a higher "value," i.e., $v_1^A v_2^A > v_1^B v_2^B$. The standard analysis of Bertrand competition implies that B sets the minimum prices $p_1^{B*} = p_2^{B*} = 0$ and A slightly undercuts B to capture the entire market. From Lemma 2, this implies

$$\frac{v_2^A + v_2^B}{N_2} p_1^{A*} + \frac{v_1^A + v_1^B}{N_1} p_2^{A*} = v_1^A v_2^A - v_1^B v_2^B.$$

Subject to the above constraint, A maximizes its profit by optimally allocating the prices on the two sides, i.e.,

$$\max_{p_1^A, p_2^A \geq 0} p_1^A N_1 + p_2^A N_2 \quad \text{s.t.} \quad \frac{v_2^A + v_2^B}{N_2} p_1^A + \frac{v_1^A + v_1^B}{N_1} p_2^A = v_1^A v_2^A - v_1^B v_2^B. \tag{2.7}$$

Solving the above problem gives us platform A's optimal pricing strategy.

Proposition 3 *Suppose $v_1^A v_2^A > v_1^B v_2^B$ and $v_1^A + v_1^B < v_2^A + v_2^B$.*[15] *Under potential maximization, stage 1 is a Bertrand equilibrium with*

$$p_1^{A*} = 0, \quad p_2^{A*} = \frac{v_1^A v_2^A - v_1^B v_2^B}{v_1^A + v_1^B} N_1, \quad p_1^{B*} = 0, \quad p_2^{B*} = 0.$$

All agents join platform A in stage 2, and A's equilibrium profit is $\pi^{A*} = \frac{v_1^A v_2^A - v_1^B v_2^B}{v_1^A + v_1^B} N_1 N_2$.

2.3.3 Discussion

As shown in Proposition 3, the market tips to the platform with the higher value of $v_1^m v_2^m$ regardless of the number of agents N_1 and N_2 on each side. Following Section 2.2.4, I discuss the three key implications under the current framework.

Divide and Conquer Similar to the monopolist in the baseline model, the dominant platform (A) always provides free access to one side and extracts surplus from the other. The weaker the competitor (in terms of the values of v_1^B and v_2^B), the more surplus the dominant platform extracts from the conquer side.

Divide/Conquer Side The divide/conquer side of the dominant platform depends on the average (or the sum of) per-interaction benefits $v_1^A + v_1^B$ and $v_2^A + v_2^B$ across the competing platforms instead of its own per-interaction benefits v_1^A and v_2^A. This implies that the decision on the divide/conquer side for the dominant platform is significantly affected by

[15] Both conditions are generic and w.l.o.g.: the former (latter) is about the two platforms (sides). If $v_1^A v_2^A = v_1^B v_2^B$, the platforms' prices are uniquely given by $p_i^{m*} = 0$ for all i, m. All agents may coordinate on either platform; both platforms make zero profit in either case.

the per-interaction benefits delivered by other competing platforms, even though the competitors' market shares are negligible. Hence, competition can reverse the divide/conquer side of the dominant platform. To illustrate, consider the following example:

$$v_1^A = 3, \quad v_2^A = 2, \quad v_1^B = 1, \quad v_2^B = 5. \tag{2.8}$$

Platform A favors side 1 more than side 2, but platform B favors side 2 much more than side 1. Suppose initially A is a monopolist. By Proposition 2, A charges side 1 and subsidizes side 2. Suppose now B enters the market. By Proposition 3, A still dominates the market under competition, but now A subsidizes side 1 and charges side 2. Note from Proposition 2 that B makes a higher profit if A and B are separate monopolists because B can extract more surplus from one side. In other words, the optimal design of a monopoly platform might not perform well under competition; this leads us to the discussion on the optimal design of competing platforms.

Optimal Design If the platforms are very competitive (say, $v_1^A v_2^A \approx v_1^B v_2^B$), the optimal design of both platforms tends to favor both sides because the one with the lower value of $v_1^m v_2^m$ has zero market share in equilibrium. By contrast, if one of them is inferior (say, $v_1^B \approx v_2^B \approx 0$), the optimal design of the superior platform tends to favor only the conquer side in order to extract more surplus from that side. When A dominates the market (i.e., $v_1^A v_2^A > v_1^B v_2^B$), the social surplus is $(v_1^A + v_2^A)N_1 N_2$; it can be less than the social surplus $(v_1^B + v_2^B)N_1 N_2$ if all agents coordinate on B instead (see (2.8) as an example). Also, the optimal design of A is likely to be socially suboptimal: if the platforms are very competitive, A's optimal design tends to maximize $v_1^A v_2^A$ instead of $v_1^A + v_2^A$; if B is inferior, it tends to favor only the conquer side.

2.4 Potential Maximization as Equilibrium Selection Criterion

This section further discusses potential maximization and compares it with popular selection criteria mentioned in the Related Literature.[16] First, in addition to microfounded and experimental justifications in the literature, I provide an intuitive justification for potential maximization. For expositional convenience, this section illustrates with the example (2.1) whenever helpful. By (2.3), the potential function $\Phi_{\mathbf{p}}$ of (2.1) is given by

	Join	Not join
Join	$1 - p_1/v_1 - p_2/v_2$	$-p_1/v_1$
Not join	$-p_2/v_2$	0

(2.9)

Now, if we view (2.9) as an *identical interest game* in which both agents share the same payoff, this game is strategically equivalent to the original game (2.1), in that both agents have the same best-response correspondence in these two games. Also, for any identical interest game, there is a generically unique Pareto-dominant equilibrium (which corresponds to the largest of the four numbers in (2.9)); moreover, arguably any reasonable equilibrium selection criterion would select this equilibrium.[17] Hence, if one expects two strategically equivalent games to have the same (or very similar) strategic behavior, he or she should also expect agents to coordinate on the potential maximizer in the original game.

2.4.1 Comparison with Monotonicity

Monotonicity is a mild and reasonable restriction on equilibria across subgames. Basically, in (2.1), it rules out the possibility that $(p_1, p_2) \le (p_1', p_2')$ but both agents do not join the nightclub in the former whereas both join in the latter. Clearly, potential maximization satisfies monotonicity. In other words, monotonicity implies that if the nightclub increases the fees (p_1, p_2) from low to high, at certain fees $(\hat{p}_1, \hat{p}_2)$ the selected equilibrium switches from (Join, Join) to (Not join, Not join). However, monotonicity says nothing about

[16] Insulating tariffs enable the use of sophisticated pricing strategies, which, strictly speaking, are not a selection criterion and therefore not compared here.

[17] But Pareto-dominated selection selects the opposite, making it a rather unconvincing selection criterion.

the values of $(\hat{p}_1, \hat{p}_2)$ and, therefore, cannot pin down a unique equilibrium. From this perspective, potential maximization strengthens monotonicity by specifying sensible and microfounded values for $(\hat{p}_1, \hat{p}_2)$: those satisfying $\hat{p}_1/v_1 + \hat{p}_2/v_2 = 1$.

2.4.2 Comparison with Pareto Dominance and Coalition-Proofness

Pareto dominance can be justified by coalition-proofness, which also selects the Pareto-dominant equilibrium if it exists. Moreover, they are the only microfounded selection criteria frequently used in the two-sided market literature. However, in the microfoundation of coalition-proofness (Bernheim et al. 1987; Moreno and Wooders 1996), all agents can freely discuss their strategies and make non-binding agreements before they simultaneously take their actions. This is at odds with what we observe in real-world two-sided markets, especially for giant platforms with many scattered users. By contrast, none of the microfoundations of potential maximization involve any form of communication among agents. It is precisely this lack of (re)assurance that urges agents to take safer actions rather than gambling on the Pareto-superior outcome in (2.1).

2.4.3 Comparison with Focality

Focality is the only popular selection criterion that can single out an equilibrium for most platform competition models, including that in Section 2.3 (but the prediction[18] differs from Proposition 3). As mentioned, focality treats platforms unequally by assuming all agents to always coordinate on a pre-specified platform whenever multiple equilibria exist. By contrast, potential maximization treats platforms equally in that payoff-irrelevant features (e.g., the name) of a platform play no role in the analysis. Focality faces an additional challenge: without a specific context, we cannot determine which platform should be the "focal" platform. By contrast, potential maximization unambiguously identifies the dominant platform. In equilibrium, the dominant platform is indeed "focal" because all

[18] It is easy to see that if, say, A is assumed to be the "focal" platform, it will optimally set $p_1^{A*} = v_1^A$ and $p_2^{A*} = v_2^A$. The exact values of p_1^{B*} and p_2^{B*} are unimportant; all agents will join A for any prices.

agents coordinate on this platform. Yet, this is an equilibrium outcome rather than an assumption from the start.

When there are multiple equilibria, agents' expectations are the key. But how these expectations are formed is equally (if not more) important. As noted in the pioneering work on network economics by Katz and Shapiro (1985, p. 439), "*...the expectations formation process remains an important element of the market to model explicitly.*" They repeated the same point subsequently (1994, p. 97), "*...the two equilibria are rather different, and one would like to have a theory that includes the factors that lead to one outcome or the other.*" Yet, this issue is largely ignored in the literature; researchers almost always take for granted that agents' expectations are exogenously given.

By contrast, potential maximization microfounds the formation of agents' expectations by endogenizing them as Lemmas 1 and 2 demonstrate. Precisely, they quantify how agents' expectations should depend on the fundamentals of the market (i.e., cross-side network effects v, numbers of agents N, etc.) and platforms' prices in a sensible manner (say, increasing v_1^A makes agents more likely to coordinate on platform A, ceteris paribus). After endogenizing agents' expectations, Proposition 3 shows that their *equilibrium expectations* depend crucially on the strengths of cross-side network effects of the competing platforms.

I thereby address the longest running debate in network economics: do network effects lead to inefficient lock-in? According to the findings in this paper, the answer is no. As shown in Proposition 3, an inferior platform delivering lower per-interaction benefits on both sides is defeated in equilibrium. This implies that quality largely explains the success of a dominant platform as repeatedly argued by Liebowitz and Margolis (1990, 1994, 1995, 1999, 2013, etc.). This argument is also supported by more recent empirical (Tellis et al. 2009a, 2009b; Gretz 2010) and experimental evidence (Hossain and Morgan 2009; Hossain et al. 2011). Now, it is further justified by the theoretical results of this paper.

2.5 Extensions

This section discusses various extensions in which every subgame remains a weighted potential game and, therefore, potential maximization remains applicable. The detailed analysis is given in the corresponding appendix; some trivial proofs are omitted.

2.5.1 Alternative Pricing Instruments

Sections 2.2 and 2.3 adopt Armstrong's framework in which agents are charged a subscription fee to join a platform. Nevertheless, potential maximization also applies to models with alternative pricing instruments such as transaction fees and two-part tariffs. If the monopoly platform in Section 2.2 uses transaction fees instead of subscription fees, the model has a unique equilibrium.[19] By contrast, multiple equilibria persist even if competing platforms use transaction fees. Hence, potential maximization can be applied to resolve the multiplicity of equilibria. In the appendix, I modify the model in Section 2.3 so that competing platforms set transaction fees instead of subscription fees. The platforms can now adjust the net per-interaction benefits of the two sides with transaction fees. This leads to several differences in the equilibrium outcome. First, the market tips to the platform with the larger sum of per-interaction benefits $v_1^m + v_2^m$ instead of the product of them $v_1^m v_2^m$. Second, the divide/conquer side depends on its own per-interaction benefits v_1^m and v_2^m instead of the average per-interaction benefits $v_1^A + v_1^B$ and $v_2^A + v_2^B$ across competing platforms. Third, the optimal design of platforms is to maximize $v_1^m + v_2^m$ regardless of how competitive the platforms are; this also maximizes social surplus. I also extend the analysis to two-part tariffs and shows that the equilibrium is identical to that of the transaction model. In other words, when both transaction and subscription fees are available, only the former are used.

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[19] In this equilibrium, the platform charges both sides the maximum transaction fees and thus extracts all agents' surplus.

2.5.2 Heterogeneous Agents, Multiple Platforms, Price Discrimination

Sections 2.2 and 2.3 deliberately study identical-agent models to (i) introduce potential maximization in the simplest way and (ii) identify first-order determinants of distinctive features in two-sided markets. Nevertheless, potential maximization also applies to heterogeneous-agent models. In the appendix, I allow agents to be heterogeneous in both their per-interaction and stand-alone benefits of joining a platform as well as their importance to agents on the other side. This encompasses Rochet and Tirole's (2006, Section 5) monopoly model, which in turn encompasses Rochet and Tirole's (2003, Section 2) and Armstrong's (2006, Section 3) models. For duopoly models, each agent can also derive different benefits at different platforms, encompassing Rochet and Tirole's (2003, Section 3) and Armstrong's (2006, Section 4) duopoly models. If each agent derives the same per-interaction benefit at any platform, the model can be further extended to any number of platforms. In addition, agents can also be given the outside option of not joining any platform. This encompasses Caillaud and Jullien's (2003, Sections 2 and 5) models. Furthermore, platforms can (perfectly or partially) price discriminate agents; still, every subgame remains a weighted potential game.

2.5.3 Same-Side Network Effects

Sometimes same-side network effects are present on one or both sides of a platform; they can be positive and increasing (e.g., peer/learning effect) or negative and decreasing (e.g., competition/congestion effect). In the appendix, I extend the baseline model with potentially non-monotonic same-side network effects on both sides and shows that every subgame remains a weighted potential game. I fully characterize the equilibrium under potential maximization for the case of positive and increasing same-side network effects on both sides. All three key implications in Section 2.2.4 carry over to this richer framework. In particular, the platform always divides and conquers, and the divide/conquer side is independent of same-side network effects.

2.5.4 General Temporal Structure

This paper follows typical two-sided market models in which all agents make their participation decisions simultaneously. Nevertheless, potential maximization also applies to more general temporal structures. In the appendix, I extend the baseline model such that the platform approaches some agents first, and then the remaining agents. This may be due to exogenous or strategic reasons. A possible exogenous reason as emphasized by Hagiu (2006) is that sometimes agents from one side (e.g., application and game developers) naturally arrive before those from the other (e.g., buyers). I show that every subgame at each stage is a weighted potential game. If $v_1 < v_2$, the platform always extracts all side-2 agents' surplus, whereas the revenue from side 1 depends on the numbers of side-1 and side-2 agents who move first/later. If the platform can freely choose whom to approach first/later, it will approach all agents from one side first, and then all agents from the other. It does not matter which side is the first side; the platform extracts all agents' surplus in either case. If the platform can only approach a few agents in advance, all the chosen agents will be from the side with fewer agents regardless of whether v_1 or v_2 is larger.

2.5.5 Infinite Number of Agents

This paper studies the more realistic finite-agent models, but potential maximization also applies to infinite-agent models. Sandholm (2001, 2009) defines potential games with continuous player sets and proves that infinite-agent potential games are the limits of convergent sequences of finite-agent potential games. To illustrate, suppose n_i is the mass (instead of the number) of side-i participants in the baseline model. Every subgame is now an infinite-agent potential game with the same potential function (2.3) because

$$u_i(n_j, p_i) - 0 = v_i \frac{\partial \Phi_{\mathbf{p}}(n_i, n_j)}{\partial n_i},$$

which is analogous to (2.4). Therefore, both frameworks essentially yield the same result.

2.6 Conclusion

This paper demonstrates how potential maximization can resolve the multiplicity of equilibria and derive novel insights into two-sided markets. As explained, this refinement is justified by many solid microfoundations in the game theory literature, widely supported by experimental evidence, and well describes agent behavior in two-sided markets. Furthermore, the paper shows that many two-sided market models are weighted potential games, and thus we can use the same selection criterion—potential maximization—for all these models. Moreover, the predictions under potential maximization match the reality well. In particular, two-sided platforms often divide and conquer, and the primary determinant of the divide/conquer side is cross-side network effects. This divide-and-conquer strategy implies that platforms are often designed to favor the conquer side much more than the divide side, which is often socially suboptimal. Besides, potential maximization is very tractable as the paper demonstrates. Given all these advantages, potential maximization is a very attractive approach to resolve the multiple equilibria issue in two-sided markets.

A natural direction for future research is to analyze other two-sided market models under potential maximization and discover more novel findings. In fact, Section 2.5 has already derived the respective potential functions for various two-sided market models. One could continue from there to identify the potential maximizer and then characterize the equilibrium in different contexts.

Not all two-sided market models are weighted potential games. For example, if agents can multihome, the model is generally not a weighted potential game. Nevertheless, potential maximization is not confined to weighted potential games; it also applies to a broader class of potential games such as *monotone potential games*.[20] Hence, extending potential maximization to two-sided market models belonging to other classes of potential games is another fruitful direction for future research.

[20] Morris and Ui (2005, Section 6) define monotone potential games. All game-theoretic justifications for potential maximization stated in the Related Literature apply to supermodular monotone potential games.

Chapter 3

Strong Network Effects Eliminate Spence Distortions

3.1 Introduction

An important contribution of Michael Spence (1975) is his theory of market failure in quality: when deciding on the level of quality, a profit-maximizing firm only cares about the (marginal) benefit of quality to the marginal consumer whereas a social planner cares about that to the average consumer. This distortion in quality is now known as the *Spence distortion*. In practice, regulators sometimes rectify this distortion through minimum quality standards and/or other incentive schemes targeting the socially optimal quality level (Sappington 2005).

In traditional markets, quality refers to product/service characteristics (henceforth *intrinsic quality*). Yet, today we are surrounded by markets with network effects,[1] i.e., the value of the good/service increases with the number of buyers/users. Thus, the number of buyers is considered as another dimension of quality in these so-called network markets.

A large literature shows that Spence distortions arise on both dimensions of quality in network markets.[2] These papers either assume *weak network effects* in which there is a

[1] Network effects arise in all platform markets including social networks (e.g., Facebook), online marketplaces (e.g., Amazon), and sharing economies (e.g., Airbnb), and in all technical standards including communication technologies (e.g., Zoom), programming languages (e.g., LaTeX), and keyboard designs (e.g., QWERTY).

[2] Lambertini and Orsini (2001) and Veiga (2018) document Spence distortions on intrinsic quality. Katz and Shapiro (1985) and Wright (2004) mention Spence distortions on the number of buyers; Weyl (2010) provides an extensive analysis on that. As emphasized by Weyl (p. 1652), "*This Spence distortion is likely more important in two-sided markets than the contexts for which it was originally conceived.*" Jullien (2012), Rysman and Wright (2014), White and Weyl (2016), and Alexandrov and Spulber (2017) also document Spence distortions on the number of buyers. Tan and Wright (2018) show that other distortions might offset the Spence distortion on the number of buyers. Veiga et al. (2017) document Spence distortions on

unique market-sharing equilibrium, or *strong network effects* in which there are multiple tipping equilibria and then specify a selection criterion to single out an equilibrium. Under weak network effects, firms compete for the marginal consumer (who is indifferent between buying from one firm or the other) as in markets without network effects. Thus, when setting the quality, each firm only cares about the benefit to the marginal consumer; Spence distortions naturally arise in the model. By contrast, under strong network effects, firms compete in the sense of "winner takes all." A classic example is the disc format war between Blu-ray and HD DVD in 2006–2008. Sony and Toshiba competed for all consumers to coordinate on their standards rather than competing for the marginal consumer; in fact, such a marginal consumer does not even exist. In this spirit, Spence distortions should be absent. The above argument applies to any number of firms and "sides."[3]

Yet, why is there an extensive literature documenting Spence distortions even under strong network effects? The crux is the choice of equilibrium selection criterion. In standard settings, this paper shows that all popular selection criteria in the network economics literature (namely, Pareto dominance, favorable/unfavorable expectations, coalition-proofness, and insulating tariffs) give rise to Spence distortions.[4] A follow-up question arises: is there a selection criterion leading to an alternative (and perhaps more plausible) prediction, in which Spence distortions are absent under strong network effects? This paper provides a positive answer: *potential maximization*.

Potential maximization is a refinement of Nash equilibrium in *potential games*, a concept introduced by Rosenthal (1973) and formalized by Monderer and Shapley (1996).[5] I will explain both concepts in Section 3.2.4. Under potential maximization and strong network

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both dimensions of quality.

[3]For monopoly markets, the outside option can be viewed as a second firm competing with the monopolist.

[4]See, for example, Caillaud and Jullien (2001), Hagiu (2006), and Jullien (2011) for favorable/unfavorable expectations; Ambrus and Argenziano (2009) for coalition-proofness; Weyl (2010) and White and Weyl (2016) for insulating tariffs. Favorable/unfavorable expectations are also called optimistic/pessimistic beliefs (e.g., Caillaud and Jullien 2003), focality (e.g., Halaburda and Yehezkel 2019), or incumbency advantage (e.g., Biglaiser et al. 2019).

[5]This refinement is justified by many theoretical and experimental studies; see Chan (2019, Related Literature) for a summary of established justifications.

effects, consumers coordinate on the less risky equilibrium,[6] which in turn depends on all consumers' valuations. Therefore, when setting the quality, the firm cares about the benefits to all (or average) consumers; the Spence distortion is absent.

Other novel quality distortions may arise under potential maximization. For example, when deciding on the quality level, the firm cares about reducing consumers' overall coordination risk rather than increasing consumers' equilibrium valuations. Therefore, the firm may oversupply (undersupply) quality relative to the social optimum if the intrinsic quality and the number of buyers are substitutes (complements) regarding the size of network benefits. Another distortion is that the firm cares more about how quality affects consumers with lower network benefits because increasing their valuations substantially reduces the overall coordination risk. To the best of my knowledge, this paper is the first to document these quality distortions.

The paper is organized as follows. Section 3.2 uses the simplest example to show that all popular selection criteria in the network economics literature lead to Spence distortions under strong network effects, whereas potential maximization does not. The model is naturally extended from Spence's (1975) monopoly model,[7] in which consumers derive an additional network benefit generated by other buyers. In this example, consumers differ only in their stand-alone benefits from buying the network good, and the quality of the good only affects stand-alone benefits. Under potential maximization, the monopolist's profit-maximizing quality level is the social optimum. Section 3.3 generalizes the example such that consumers differ in both their network and stand-alone benefits, and the quality of the good affects both benefits. Several novel quality distortions (but not the Spence distortion) now arise under potential maximization. Section 3.4 slightly modifies the model in Section 3.3 by endogenizing the number of potential consumers as the "quality" of the network good. I show that the Spence distortion on this dimension of quality is absent only under

[6]Potential maximization always selects the risk-dominant equilibrium (Harsanyi and Selten 1988) in two-player two-action games.

[7]Spence (1975) analyzes a monopoly model, but his insights carry over to any number of firms. Analogously, for expositional convenience, I analyze a one-sided monopoly model, but my insights carry over to any number of firms and sides.

potential maximization. Section 3.5 concludes.

3.2 An Example

The model in this section is a special case of that in Section 3.3. All the proofs are also special cases of those in the general model, and therefore I will not repeat in this section.

3.2.1 Model

A monopolist sells a network good to N consumers. Let $I \equiv \{1, \ldots, N\}$ denote the set of consumers. The game has two stages. In stage 1, the monopolist sets price $p \in \mathbb{R}_+$ and quality $q \in \mathbb{R}_+$. In stage 2, all consumers simultaneously decide whether to buy the good. Denote $a_i \in \{0 \equiv \text{not buy}, 1 \equiv \text{buy}\}$ as consumer i's action, $\mathbf{a} \equiv (a_i)_i$ as the action profile, and $\mathbf{a}_{-i} \equiv (a_j)_{j \neq i}$ as the action profile except a_i. In the context of platform markets, the monopolist is a platform providing a service, and consumers decide whether to join the platform. Consumer i's payoff is

$$u_i(a_i = 1, \mathbf{a}_{-i}, p, q) = v \sum_{j \neq i} a_j + \beta(i, q) - p, \quad u_i(a_i = 0, \mathbf{a}_{-i}, p, q) = 0,$$

where $v \in \mathbb{R}_{++}$ is the network benefit generated by every other buyer and $\beta : I \times \mathbb{R}_+ \to \mathbb{R}_+$ measures each consumer's stand-alone benefit from buying the good. In this example, consumers differ only in their stand-alone benefits, and the quality of the good affects stand-alone benefits only. These restrictions are relaxed in Section 3.3. The monopolist's payoff is equal to its profit:

$$\pi(\mathbf{a}, p, q) = p \sum_i a_i - c(q), \tag{3.1}$$

where $c : \mathbb{R}_+ \to \mathbb{R}_+$ measures the cost of quality.[8] Note that if $v = 0$ (which I have ruled out), this example is essentially the Spence's (1975) model; see also Sheshinski (1976) for a similar model.

I make the following assumptions:

[8] We can easily generalize the cost function to also depend on the number of buyers. All insights remain intact; see the appendix for details.

A1. $c(q)$ is increasing, strictly convex, differentiable, $\lim_{q\to 0} c'(q) = 0$, and $\lim_{q\to\infty} c'(q) = \infty$;

A2. for all $i \in I$, $\beta(i, q)$ is increasing, concave, and differentiable in q;

A3. for all $i \in \{1, \dots, N-1\}$ and $q \in \mathbb{R}_+$, $0 \le \beta(i, q) - \beta(i+1, q) < v$.

A1 and A2 are standard assumptions to guarantee a unique interior solution for both the socially optimal and equilibrium quality levels in the subsequent analysis. A3 states that (i) consumers are well-ordered in their stand-alone benefits (consumer 1 (N) always values the good most (least)) and (ii) the network benefit v is large relative to the heterogeneity of consumers' stand-alone benefits.[9]

In what follows, I examine the pure-strategy subgame-perfect equilibria of this two-stage game. Section 3.2.2 defines the social planner's problem and derives the socially optimal quality level. Section 3.2.3 analyzes the game under all popular selection criteria and shows that the Spence distortion arises in equilibrium. Section 3.2.4 introduces potential games and potential maximization and shows that the Spence distortion is absent under potential maximization.

3.2.2 Social Planner's Problem

The social planner chooses quality q and all consumers' purchasing decisions $\mathbf{a}$ to maximize social surplus, which is given by

$$S(\mathbf{a}, q) = \sum_i a_i(v \sum_{j \neq i} a_j + \beta(i, q)) - c(q).$$

The first term is the sum of all consumers' benefits, and the second term is the cost of quality. Given the marginal cost of production is zero, the social planner assigns all

[9] Given any $q \in \mathbb{R}_+$, we can always permute consumers in decreasing order of their stand-alone benefits. Thus, A3 can be relaxed as follows: (i) given any $q \in \mathbb{R}_+$, after the permutation, $\beta(i, q) - \beta(i+1, q) < v$ for all $i \in \{1, \dots, N-1\}$ and (ii) for all $q \in \mathbb{R}_+$, $N \in \arg\min \beta(i, q)$. The latter guarantees that the Spence distortion is well-behaved under all popular selection criteria because consumer N is always the marginal consumer in equilibrium. If we are only interested in potential maximization, the latter can be dropped.

consumers to buy the good. Thus, social surplus becomes

$$S(\mathbf{1}, q) = vN(N-1) + \sum_i \beta(i, q) - c(q). \tag{3.2}$$

Under A1 and A2, the unique socially optimal quality level q^{FB} (FB stands for "First Best") is characterized by the first-order condition of (3.2) as shown below.

Lemma 1 *The socially optimal quality level q^{FB} is uniquely given by*

$$c'(q^{FB}) = \sum_i \frac{\partial \beta(i, q^{FB})}{\partial q}.$$

In words, the social optimum equates the marginal cost of quality to the sum of all consumers' marginal values of quality.

3.2.3 Monopolist's Problem under All Popular Selection Criteria

I now turn to the monopolist's problem. I solve the game backwards, starting from stage 2. Strong network effects often generate multiple equilibria in stage 2. In particular, there are exactly two pure-strategy Nash equilibria when $\beta(1, q) \le p \le v(N-1) + \beta(N, q)$:[10] (i) all consumers buy the good and (ii) no one buys the good. Clearly, the former Pareto dominates the latter for all consumers. All popular selection criteria in this literature (i.e., Pareto dominance, favorable/unfavorable expectations, coalition-proofness, and insulating tariffs; see footnote 4 for examples) except unfavorable expectations select the former whenever there are multiple equilibria, whereas unfavorable expectations select the latter instead. I now analyze these two scenarios. For convenience, I call the first scenario favorable expectations.

Favorable expectations Under favorable expectations, I prove in the appendix that, for any quality q chosen by the monopolist, its optimal price is

$$p^*(q) = v(N-1) + \beta(N, q).$$

[10] The unique (and dominant-solvable) equilibrium is all consumers buying the good when $p < \beta(1, q)$ and no one buying the good when $p > v(N-1) + \beta(N, q)$. See the appendix for details.

All consumers buy the good in stage 2, and consumer N—the marginal consumer—is indifferent between buying or not. Given this optimal pricing strategy, from (3.1), the monopolist's profit is

$$\pi(\mathbf{1}, p^*(q), q) = vN(N-1) + N\beta(N, q) - c(q).$$

Under A1 and A2, its unique profit-maximizing quality level q^* is characterized by the first-order condition of the above expression as shown below.

Lemma 2 *Under favorable expectations, the equilibrium quality level q^* is uniquely given by*

$$c'(q^*) = N\frac{\partial\beta(N, q^*)}{\partial q}.$$

In words, the equilibrium quality level equates the marginal cost of quality with the marginal value of quality to the marginal consumer multiplied by the number of consumers. The difference between the social optimum q^{FB} in Lemma 1 and the equilibrium quality level q^* in Lemma 2 in such a way is known as the *Spence distortion.*

Unfavorable expectations Under unfavorable expectations, the monopolist can at most (and will) charge

$$p^*(q) = \beta(1, q), \tag{3.3}$$

so that all consumers will buy the good in stage 2 for sure.[11] No one will buy the good if it charges a higher price. Hence, from (3.1), its profit becomes

$$\pi(\mathbf{1}, p^*(q), q) = N\beta(1, q) - c(q). \tag{3.4}$$

Under A1 and A2, its unique profit-maximizing quality level q^* is characterized by the first-order condition of (3.4) as shown below.

[11] The unique equilibrium is all consumers buying the good when $p < \beta(1, q)$ as stated in footnote 10.

Lemma 3 *Under unfavorable expectations, the equilibrium quality level q^* is uniquely given by*

$$c'(q^*) = N\frac{\partial\beta(1,q^*)}{\partial q}.$$

Similar but distinct from the Spence distortion under favorable expectations as shown in Lemma 2, the monopolist now only cares about the benefit of quality to the consumer with the highest valuation (i.e. consumer 1) under unfavorable expectations.

3.2.4 Monopolist's Problem under Potential Maximization

Now, I first introduce potential games and potential maximization, and then I will analyze the game under potential maximization. A game is called a *potential game* if it admits a *potential function*, which summarizes all consumers' strategic considerations. Consider the following function

$$\Phi(\mathbf{a},p,q) = \frac{v}{2}\sum_i (a_i \sum_{j\neq i} a_j) + \sum_i a_i\beta(i,q) - p\sum_i a_i. \tag{3.5}$$

Given any price p and quality q, we can easily verify that for all $i \in I$ and $\mathbf{a}_{-i} \in \{0,1\}^{N-1}$,

$$u_i(a_i = 1, \mathbf{a}_{-i}, p, q) - u_i(a_i = 0, \mathbf{a}_{-i}, p, q) = \Phi(a_i = 1, \mathbf{a}_{-i}, p, q) - \Phi(a_i = 0, \mathbf{a}_{-i}, p, q). \tag{3.6}$$

In words, fixing others' actions $\mathbf{a}_{-i}$, the payoff difference between buying the good or not for each agent is equal to the corresponding difference in Φ. Such a function Φ is called a potential function. Hence, every subgame with (p, q) set by the monopolist in stage 1 is a potential game.

The maximizer of the potential function (i.e., $\max_{\mathbf{a}} \Phi(\mathbf{a}, p, q)$; also called the *potential maximizer*) always exists and is generically unique. Furthermore, the potential maximizer is a Nash equilibrium: if someone deviates from the potential maximizer, the potential will decrease, and, by (3.6), the deviator will have a lower payoff. Hence, if there is a unique equilibrium in a potential game, the potential maximizer is the unique equilibrium; if there are multiple equilibria, the potential maximizer is the one with the highest potential. Moreover, as stated in footnote 5, many theoretical and experimental studies find that players often coordinate on the potential maximizer. Therefore, the potential maximizer refines Nash equilibrium in potential games, and this refinement is called *potential maximization.*

I now analyze the game under potential maximization. As stated on p. 52, there are two equilibria in stage 2 when $\beta(1,q) \leq p \leq v(N-1) + \beta(N,q)$. By (3.5), their respective potentials are

$$\Phi(\mathbf{1}, p, q) = \frac{v}{2}N(N-1) + \sum_i \beta(i,q) - pN, \quad \Phi(\mathbf{0}, p, q) = 0.$$

Clearly, the potential maximizer is all consumers buying the good if and only if $p \leq \frac{v}{2}(N-1) + \frac{1}{N}\sum_i \beta(i,q)$ for these subgames. The following lemma shows that this finding holds for all subgames.

Lemma 4 *The potential maximizer of the subgame is all consumers buying the good if* $p \leq \frac{v}{2}(N-1) + \frac{1}{N}\sum_i \beta(i,q)$ *and no one buying the good otherwise.*

Proof. See the appendix. ■

As shown in Lemma 4, the monopolist has to leave enough consumer surplus by setting sufficiently low price p and/or high quality q in stage 1 so that all consumers will buy the good in stage 2. Thus, the monopolist optimally sets the highest possible price until the constraint binds, i.e.,

$$p^*(q) = \frac{v}{2}(N-1) + \frac{1}{N}\sum_i \beta(i,q).$$

Given this optimal pricing strategy, from (3.1), its profit is

$$\pi(\mathbf{1}, p^*(q), q) = \frac{v}{2}N(N-1) + \sum_i \beta(i,q) - c(q).$$

Observe that this profit function is identical to the social surplus (3.2) except the network benefit v is now halved. Therefore, the monopolist's choice of quality is the same as that of a social planner, and it is summarized as follows.

Proposition 1 *Under potential maximization, the equilibrium quality level is socially optimal: the Spence distortion is absent.*

In this example, the monopolist chooses the socially optimal quality level in equilibrium. This implies that regulating the monopolist can only reduce social surplus. Nevertheless, this implication may not hold when we generalize the model in Section 3.3. In the general model, some novel quality distortions (but not the Spence distortion) may arise under potential maximization. To the best of my knowledge, these distortions are never documented in the literature.

3.3 Generalization

In the previous section, consumers differ only in their stand-alone benefits, and the quality of the good only affects stand-alone benefits. This section relaxes these restrictions and analyzes a broader class of payoff functions.

3.3.1 Model

The model is identical to the previous one except the payoff of consumer i from buying the good is now

$$u_i(a_i = 1, \mathbf{a}_{-i}, p, q) = v(i, q) \cdot \alpha(\sum_j a_j, q) + \beta(i, q) - p,$$

where $\alpha : I \times \mathbb{R}_+ \to \mathbb{R}_+$ is the network-benefit function and $v : I \times \mathbb{R}_+ \to \mathbb{R}_{++}$ measures each consumer's valuation of the network benefit. Consumers now differ in two aspects: (i) their network-benefit valuations v and (ii) their stand-alone benefits β. Quality now affects their payoffs in three aspects: (i) their network-benefit valuations v, (ii) the network-benefit function α, and (iii) their stand-alone benefits β. Clearly, the previous example is a special case with $v(i, q) = v$ and $\alpha(\sum_j a_j, q) = \sum_j a_j - 1 = \sum_{j \neq i} a_j$. In the appendix, I further generalize the model with multidimensional quality and a general cost function for the monopolist that also depends on the number of buyers. All insights developed in this section carry over to this even more general framework.

I make the following assumptions. These assumptions boil down to A1–A3 when $v(i, q) = v$ and $\alpha(\sum_j a_j, q) = \sum_j a_j - 1$.

B1. $c(q)$ is increasing, sufficiently convex, differentiable, and $\lim_{q\to 0} c'(q) = 0$;

B2. for all $i \in I$, $v(i,q)$, $\alpha(i,q)$, and $\beta(i,q)$ are increasing and differentiable in q;

B3. for all $q \in \mathbb{R}_+$, $v(i,q)$ and $\beta(i,q)$ are decreasing in i;

B4. for all $q \in \mathbb{R}_+$, $\alpha(1,q) = 0$ and $v(i,q)\alpha(i,q) + \beta(i,q)$ is strictly increasing in i.

B1 and B2 guarantee a unique interior solution for both the socially optimal and equilibrium quality levels. B3 states that consumers are well-ordered on both their network and stand-alone benefits.[12] B4 states that (i) the network benefit, by definition, is zero when there is only one buyer and (ii) the marginal network benefit of additional buyers is large relative to the heterogeneity of consumers' valuations.

Similar to Section 3.2, Section 3.3.2 derives the socially optimal quality level. Section 3.3.3 analyzes the game under all popular selection criteria and shows that Spence distortions arise in equilibrium. Section 3.3.4 shows that Spence distortions are absent but other quality distortions arise under potential maximization.

3.3.2 Social Planner's Problem

Social surplus is now given by

$$S(\mathbf{a}, q) = \sum_i a_i[v(i,q)\alpha(\sum_j a_j, q) + \beta(i,q)] - c(q).$$

Given the marginal cost of production is zero, the social planner assigns all consumers to buy the good. Thus, social surplus becomes

$$S(\mathbf{1}, q) = \sum_i v(i,q)\alpha(N,q) + \sum_i \beta(i,q) - c(q). \tag{3.7}$$

---

[12] Similar to Section 3.2 (see also footnote 9), consumers need not be well-ordered (i.e., B3 can be relaxed) as long as the marginal network benefit is sufficiently large (i.e., by strengthening B4). Precisely, we can replace B3 and B4 with the following assumption: for all $n \in \{1, \ldots, N-1\}$ and $q \in \mathbb{R}_+$, $\alpha(1,q) = 0$ and $\max_i \{v(i,q)\alpha(n,q) + \beta(i,q)\} < \min_i \{v(i,q)\alpha(n+1,q) + \beta(i,q)\}$. To guarantee that Spence distortions are well-behaved under favorable expectations, we need an additional assumption: for all $q \in \mathbb{R}_+$, $N \in \arg\min_i \{v(i,q)\alpha(N,q) + \beta(i,q)\}$.

Under B1 and B2, the unique socially optimal quality level q^{FB} is characterized by the first-order condition of (3.7) as shown below.

Lemma 5 *The socially optimal quality level q^{FB} is uniquely given by*

$$c'(q^{FB}) = \alpha(N, q^{FB}) \sum_i \frac{\partial v(i, q^{FB})}{\partial q} + \sum_i v(i, q^{FB}) \frac{\partial \alpha(N, q^{FB})}{\partial q} + \sum_i \frac{\partial \beta(i, q^{FB})}{\partial q}.$$

As in Section 3.2.2, the social optimum equates the marginal cost of quality with the sum of the marginal values of quality for all consumers.

3.3.3 Monopolist's Problem under All Popular Selection Criteria

I now analyze the monopolist's problem. There are two equilibria in stage 2 when $\beta(1, q) \leq p \leq v(N, q)\alpha(N, q) + \beta(N, q)$:[13] (i) all consumers buy the good and (ii) no one buys the good. Similar to Section 3.2.3, I analyze the game under both favorable and unfavorable expectations.

Favorable expectations Under favorable expectations, I prove in the appendix that, for any quality q chosen by the monopolist, its optimal price is

$$p^*(q) = v(N, q)\alpha(N, q) + \beta(N, q). \tag{3.8}$$

All consumers buy the good in stage 2, and the marginal consumer N is indifferent between buying or not. Given this optimal pricing strategy, from (3.1), its profit is

$$\pi(\mathbf{1}, p^*(q), q) = Nv(N, q)\alpha(N, q) + N\beta(N, q) - c(q). \tag{3.9}$$

Under B1 and B2, its unique profit-maximizing quality level q^* is characterized by the first-order condition of (3.9) as shown below.

[13] The unique (and dominant-solvable) equilibrium is all consumers buying the good when $p < \beta(1, q)$ and no one buying the good when $p > v(N, q)\alpha(N, q) + \beta(N, q)$. See the appendix for details.

Lemma 6 *Under favorable expectations, the equilibrium quality level q^* is uniquely given by*

$$c'(q^*) = \alpha(N, q^*) \underbrace{N\frac{\partial v(N, q^*)}{\partial q}}_{Spence\ distortion} + \underbrace{Nv(N, q^*)}_{Spence\ distortion} \frac{\partial \alpha(N, q^*)}{\partial q} + \underbrace{N\frac{\partial \beta(N, q^*)}{\partial q}}_{Spence\ distortion}.$$

In contrast to the social optimum in Lemma 5, the equilibrium quality level equates the marginal cost of quality with the marginal value of quality to the marginal consumer multiplied by the number of consumers. Spence distortions arise under favorable expectations.

Unfavorable expectations Under unfavorable expectations, it is easy to see that the monopolist's optimal pricing strategy is given by (3.3), i.e., the same price as in Section 3.2.3. It is because the monopolist cannot internalize any network benefits under unfavorable expectations, and therefore the generalizations in this section play no role in this scenario. All subsequent analysis follows.

3.3.4 Monopolist's Problem under Potential Maximization

Similar to Section 3.2.4, consider the following function

$$\Phi(\mathbf{a}, p, q) = \sum_{i=1}^{\sum_j a_j} \alpha(i, q) + \sum_i a_i \frac{\beta(i, q)}{v(i, q)} - p \sum_i \frac{a_i}{v(i, q)}. \tag{3.10}$$

Given any p and q, we can verify that for all i and $\mathbf{a}_{-i}$,

$$u_i(a_i = 1, \mathbf{a}_{-i}, p, q) - u_i(a_i = 0, \mathbf{a}_{-i}, p, q) = v(i, q)[\Phi(a_i = 1, \mathbf{a}_{-i}, p, q) - \Phi(a_i = 0, \mathbf{a}_{-i}, p, q)].$$

In other words, the payoff difference between buying the good or not for each agent is proportional to the corresponding difference in Φ. Every subgame is now called a *weighted potential game* (if the proportion is one for all consumers as in (3.6), then it is called an *exact potential game*). Nevertheless, the potential function Φ preserves all the properties stated in Section 3.2.4. The potential maximizer of each subgame is given as follows.

Lemma 7 *The potential maximizer of the subgame is all consumers buying the good if*

$$p \leq \frac{\sum_i \alpha(i,q) + \sum_i \frac{\beta(i,q)}{v(i,q)}}{\sum_i \frac{1}{v(i,q)}}, \tag{3.11}$$

and no one buying the good otherwise.

Proof. See the appendix. ■

As shown in Lemma 7, the monopolist has to leave enough surplus to consumers so that all of them will buy the good. Therefore, it optimally sets the highest possible price until the constraint in (3.11) binds. By rearranging the terms, its optimal pricing strategy is characterized as follows.

Proposition 2 *For any quality q set by the monopolist, its optimal pricing strategy under potential maximization is*

$$p^*(q) = \underbrace{\left(\frac{1}{N}\sum_i \frac{1}{v(i,q)}\right)^{-1}}_{harmonic\ mean} \times \underbrace{\frac{1}{N}\sum_i \alpha(i,q)}_{incremental\ benefit} + \sum_i \underbrace{\frac{\frac{1}{v(i,q)}}{\sum_j \frac{1}{v(j,q)}}}_{bias\ weights} \beta(i,q).$$

In contrast to the equilibrium price under favorable expectations (3.8) or unfavorable expectations (3.3), the price now depends on all consumers' valuations but not that of a single consumer. Clearly, Spence distortions will not arise in equilibrium. Nevertheless, other quality distortions will arise. Note that the monopolist's equilibrium price (and thus its profit-maximizing quality level) depends on consumers' overall coordination risk, which in turn depends on the three terms comprising $p^*(q)$ in Proposition 2. These three terms are the sources of quality distortions in the subsequent analysis. The larger these terms, the lower the coordination risk, and thus the more the monopolist can charge consumers. I now discuss these three terms one by one.

Harmonic mean The first term is the harmonic mean of consumers' network-benefit valuations, denoted by

$$H(v|q) \equiv \left(\frac{1}{N}\sum_i \frac{1}{v(i,q)}\right)^{-1}. \tag{3.12}$$

Compared to the arithmetic mean, the harmonic mean alleviates the impact of large outliers and exacerbates that of small ones. Also, it is always less than the arithmetic mean and decreases with a mean-preserving spread. In other words, the monopolist internalizes only part of consumers' network-benefit valuations; the more dispersed their network-benefit valuations, the less surplus the monopolist can extract.

Incremental benefit The second term $\frac{1}{N}\sum_i \alpha(i,q)$ is called the incremental (network) benefit as suggested by its form. Under potential maximization, all consumers will buy the good in equilibrium, and therefore the equilibrium network benefit is $\alpha(N,q)$. However, the monopolist internalizes only part of the network benefit. For a fixed equilibrium network benefit $\alpha(N,q)$, a larger incremental benefit implies a lower overall coordination risk,[14] and thus the monopolist can charge consumers more.

Bias weights The last term is a weighted average of consumers' stand-alone benefits. Consumers with lower network-benefit valuations relatively care more about the stand-alone benefit from buying the good. Thus, the stand-alone benefits of these consumers are relatively more important regarding the overall coordination risk. When consumers with lower network-benefit valuations also have lower (higher) stand-alone benefits, it is more (less) difficult for all consumers to coordinate on buying the good, and thus the monopolist's equilibrium price is lower (higher).

Given the monopolist's optimal pricing strategy in Proposition 2, from (3.1), its profit is

$$\pi(\mathbf{1}, p^*(q), q) = \underbrace{NH(v|q)}_{N\times\text{harmonic mean}} \times \underbrace{\frac{1}{N}\sum_i \alpha(i,q)}_{\text{incremental benefit}} + \sum_i \underbrace{\frac{\frac{1}{v(i,q)}}{\frac{1}{N}\sum_j \frac{1}{v(j,q)}}}_{\text{bias weights}} \beta(i,q) - c(q). \tag{3.13}$$

This profit function differs from the social surplus (3.7) in three aspects:

[14] When the incremental benefit is large, even if only some (but not all) consumers buy the good, the network benefit is still large, i.e., the cost of miscoordination is small.

1. the sum of consumers' network-benefit valuations $\sum_i v(i,q)$ is replaced by the harmonic mean of the valuations $H(v|q)$ multiplied by the number of consumers N;

2. the equilibrium network benefit $\alpha(N,q)$ is replaced by the incremental network benefit $\frac{1}{N}\sum_i \alpha(i,q)$;

3. the sum of consumers' stand-alone benefits $\sum_i \beta(i,q)$ is replaced by a weighted sum of the benefits.

These three differences lead to various quality distortions (but not Spence distortions). Under B1 and B2, the monopolist's unique profit-maximizing quality level q^* is characterized by the first-order condition of (3.13) as shown below.

Proposition 3 *Under potential maximization, the equilibrium quality level q^* is uniquely given by*

$$
\begin{aligned}
& c'(q^*) \\
= \; & \underbrace{\sum_i \left(\frac{\frac{1}{v(i,q^*)}}{\sum_j \frac{1}{v(j,q^*)}} \frac{p^*(q^*) - \beta(i,q^*)}{v(i,q^*)\alpha(N,q^*)} \right) \alpha(N,q^*)}_{\textit{adjusted-incremental-benefit distortion}} \times \sum_i \underbrace{\frac{\frac{1}{v(i,q^*)} \frac{p^*(q^*)-\beta(i,q^*)}{v(i,q^*)\alpha(N,q^*)}}{\frac{1}{N}\sum_j \left(\frac{1}{v(j,q^*)} \frac{p^*(q^*)-\beta(j,q^*)}{v(j,q^*)\alpha(N,q^*)} \right)}}_{\textit{bias-weights distortion } (v)} \frac{\partial v(i,q^*)}{\partial q} \\
+ \; & \underbrace{NH(v|q^*)}_{\textit{harmonic-mean distortion}} \times \underbrace{\frac{1}{N}\sum_i \frac{\partial \alpha(i,q^*)}{\partial q}}_{\textit{marginal-incremental-benefit distortion}} + \sum_i \underbrace{\frac{\frac{1}{v(i,q^*)}}{\frac{1}{N}\sum_j \frac{1}{v(j,q^*)}}}_{\textit{bias-weights distortion } (\beta)} \frac{\partial \beta(i,q^*)}{\partial q} .
\end{aligned}
$$

Compared to the social optimum in Lemma 5, there are several quality distortions affecting consumers' valuations for (i) the network benefit, (ii) the network-benefit function, and (iii) consumers' stand-alone benefits. I now discuss them one by one, starting from the last one to the first one.

Stand-alone benefits The distortion affecting stand-alone benefits is called the *bias-weights distortion* (β). As its name suggests, the monopolist assigns higher weights to consumers with lower network-benefit valuations because these consumers' stand-alone benefits are relatively more important regarding the overall coordination risk—the same reason we have discussed previously on the bias weights of the monopolist's equilibrium price. If the quality of the good only affects stand-alone benefits and consumers have the same network-benefit valuation (as in Section 3.2), the equilibrium quality level is socially optimal (as shown in Proposition 1).

Network-benefit function There are two distortions affecting the network-benefit function. The first one is called the *harmonic-mean distortion*. As shown in Proposition 2, the monopolist's equilibrium price increases linearly with $H(v|q)$ (the harmonic mean), which is less than $\frac{1}{N}\sum_i v(i,q)$ (the arithmetic mean) as explained before. Therefore, the monopolist internalizes only part of consumers' network-benefit valuations, giving it too little incentive to increase the network-benefit function by investing in quality. The second distortion is called the *marginal-incremental-benefit distortion*. As shown in Proposition 2, the monopolist's equilibrium price increases linearly with the incremental benefit $\frac{1}{N}\sum_i \alpha(i,q)$. Therefore, when setting the quality, the monopolist cares about increasing the incremental benefit rather than increasing the equilibrium network benefit $\alpha(N,q)$. Thus, the monopolist tends to oversupply (undersupply) quality if the quality and the number of buyers are substitutes (complements) regarding the network benefit, i.e., if $\frac{\partial\alpha(i,q)}{\partial q}$ decreases (increases) with i.

Network-benefit valuations There are two distortions affecting consumers' valuations for the network benefit. The first one is called the *adjusted-incremental-benefit distortion*.[15] The term $p^*(q)-\beta(i,q)$ is consumer i's net stand-alone cost from buying the good, and $v(i,q)\alpha(N,q)$ is his network benefit if all consumers successfully coordinate on buying the good. Hence, $\frac{p^*(q)-\beta(i,q)}{v(i,q)\alpha(N,q)}$ can be viewed as the cost-benefit ratio from taking his risky action (i.e., buying the good). Clearly, all consumers' ratios (and thus the weighted average

[15] As we will see shortly in Corollary 2, this distortion is simplified to $\frac{1}{N}\sum_i \alpha(i,q)$ (called the incremental-benefit distortion) when consumers have the same network-benefit valuation. Therefore, the current distortion is called the adjusted-incremental-benefit distortion.

of their ratios $\sum_i \frac{\frac{1}{v(i,q)}}{\sum_j \frac{1}{v(j,q)}} \frac{p^*(q)-\beta(i,q)}{v(i,q)\alpha(N,q)})$ are positive and smaller than one,[16] indicating the monopolist's insufficient incentive to increase consumers' network-benefit valuations by investing in quality. The second distortion is called the *bias-weights distortion* (v). As its name suggests, the monopolist assigns higher weights to consumers with higher cost-benefit ratios $\frac{p^*(q)-\beta(i,q)}{v(i,q)\alpha(N,q)}$ and/or lower network-benefit valuations $v(i,q)$ because increasing their $v(i,q)$ substantially reduces the overall coordination risk.[17]

Proposition 3 identifies five quality distortions under potential maximization. Both the adjusted-incremental-benefit and harmonic-mean distortions put downward pressures on the equilibrium quality level, while that of the marginal-incremental-benefit distortion is upward (downward) when i and q are substitutes (complements) in the network-benefit function $\alpha(i,q)$. The effects of the two bias-weights distortions are ambiguous. Nevertheless, they are more likely to be downward rather than upward pressures because consumers who value the good less are usually less sensitive to the marginal benefit of quality. Overall speaking, the monopolist is more likely to undersupply rather than oversupply quality relative to the social optimum.

This section analyzes the general model in which consumers are heterogeneous in two aspects (v and β) and the quality affects their payoffs in three aspects (v, α, and β). For a specific network market, only some of these aspects are relevant and important, and thus only some of the five distortions will arise. I conclude this section with two corollaries characterizing the distortions for two special cases of interest: (i) $\beta(i,q) = 0$ and (ii) $v(i,q) = v(q)$ for all $i \in I$. The former fits network markets that create value primarily by enabling interactions among users (e.g., telecommunications and social networks).

[16] In the appendix, I prove that $\beta(1,q) < p^*(q) < v(N,q)\alpha(N,q) + \beta(N,q)$, which (together with B3) implies all consumers' ratios are positive and smaller than one. Intuitively, if the ratio is negative (larger than one) for a consumer, he has a dominant (dominated) strategy to buy the good, which contradicts our equilibrium analysis.

[17] Consumers with higher cost-benefit ratios are less willing to take their risky actions (i.e., buying the good), and thus increasing their $v(i,q)$ makes it much easier for all consumers to coordinate on buying the good. For consumers with lower network-benefit valuations, increasing their $v(i,q)$ substantially increases the monopolist's equilibrium price (which increases linearly with $H(v|q)$) because the harmonic mean exacerbates the impact of small outliers as explained before.

The latter does not perfectly fit most of the markets that come to mind, but it substantially simplifies the distortions because the complexity due to the harmonic mean of $v(i,q)$ disappears.

Corollary 1 *When $\beta(i,q)=0$ for all $i \in I$, the equilibrium quality level q^* under potential maximization is uniquely given by*

$$c'(q^*) \;=\; \underbrace{\left(1+\frac{Var(v^{-1}|q^*)}{(E(v^{-1}|q^*))^2}\right)}_{dispersion\ distortion} \times \underbrace{\frac{1}{N}\sum_i \alpha(i,q^*)}_{incremental\text{-}benefit\ distortion} \times \sum_i \underbrace{\frac{\frac{1}{v(i,q^*)^2}}{\frac{1}{N}\sum_j \frac{1}{v(j,q^*)^2}}}_{bias\text{-}weights\ distortion\ (v)} \frac{\partial v(i,q^*)}{\partial q}$$
$$+ \underbrace{NH(v|q^*)}_{harmonic\text{-}mean\ distortion} \times \underbrace{\frac{1}{N}\sum_i \frac{\partial \alpha(i,q^*)}{\partial q}}_{marginal\text{-}incremental\text{-}benefit\ distortion}.$$

When there are no stand-alone benefits, the distortions affecting the network-benefit function remain intact, and the term for the bias-weights distortion (v) is simplified. The adjusted-incremental-benefit distortion now breaks into two distortions: the *dispersion distortion* and the *incremental-benefit distortion.* Let $E(v^{-1}|q) \equiv \frac{1}{N}\sum_i \frac{1}{v(i,q)}$ and $Var(v^{-1}|q) \equiv \frac{1}{N}\sum_i(\frac{1}{v(i,q)} - E(v^{-1}|q))^2$ denote the arithmetic mean and the variance of $\frac{1}{v(i,q)}$ respectively. As shown in Corollary 1, the dispersion distortion puts an upward pressure on quality, and it increases with the mean-preserving spread of $\frac{1}{v(i,q)}$. Thus, the dispersion distortion counteracts the harmonic-mean distortion: the former increases with the dispersion of consumers' network-benefit valuations, while the latter decreases with it. On the other hand, the incremental-benefit distortion puts a downward pressure on quality because the incremental network benefit $\frac{1}{N}\sum_i \alpha(i,q)$ is less than the equilibrium network benefit $\alpha(N,q)$. Given that the dispersion and incremental-benefit distortions constitute the adjusted-incremental-benefit distortion, their net effect is a downward pressure on quality.

Corollary 2 *When $v(i,q) = v(q)$ for all $i \in I$, the equilibrium quality level q^* under* *potential maximization is uniquely given by*

$$c'(q^*) = \underbrace{\frac{1}{N}\sum_i \alpha(i,q^*)}_{\textit{incremental-benefit distortion}} \times Nv'(q^*) + \quad Nv(q^*) \times \underbrace{\frac{1}{N}\sum_i \frac{\partial\alpha(i,q^*)}{\partial q}}_{\textit{marginal-incremental-benefit distortion}} + \sum_i \quad \frac{\partial\beta(i,q^*)}{\partial q}.$$

When consumers have the same network-benefit valuation, the two bias-weights and harmonic-mean distortions disappear. The marginal-incremental-benefit distortion remains intact, and the adjusted-incremental-benefit distortion is simplified to the incremental-benefit distortion.

As shown in Proposition 3 and Corollaries 1 and 2, quality distortions caused by the incremental benefit are robust to the heterogeneity of consumers' valuations. This suggests that when setting policies and regulations for network markets, arguments based on these distortions are often valid, whereas those for other distortions (especially the two bias-weights distortions) require considerable knowledge of consumers' preferences.

3.4 Number of Buyers as Quality

The previous sections strictly follow Spence's (1975) model in which the monopolist sets both price p and quality q. As mentioned in the Introduction, the number of buyers/users is considered as another dimension of quality in network markets, and the Spence distortion on this dimension is also widely documented in the literature (see footnote 2). As emphasized by Weyl (2010, p. 1652), "*This Spence distortion is likely more important in two-sided markets than the contexts for which it was originally conceived.*" At this point, it should be apparent that strong network effects will eliminate this Spence distortion under potential maximization for the same reason as before. This section formally proves this result by slightly modifying the model in Section 3.3 and derives further implications.

3.4.1 Model

In the previous model, the number of consumers is fixed at N. This section endogenizes the number of consumers, which serves as the "quality" of the network good. Precisely, I add an additional stage (stage 0) in the beginning of the game where the monopolist chooses the number of consumers $N \in \mathbb{Z}_+$ through some costly efforts.[18] These efforts include promotion and advertising efforts, and expanding demographic and geographic market coverage. For expositional convenience, I remove the intrinsic quality q from the model. The monopolist's payoff is now

$$\pi(\mathbf{a}, p, N) = p \sum_i a_i - c(N),$$

where $c : \mathbb{Z}_+ \to \mathbb{R}_+$ measures the cost.

I replace B1 and B2 with the following assumption:

C1. $c(N)$ is increasing, sufficiently convex, and $c(0) = c(1)$.

C1 and B4 guarantee a unique interior solution for both the socially optimal and equilibrium numbers of consumers in the subsequent analysis. B3 is a natural assumption in the current setting because consumers who value the good more are often better informed about the existence of the good.

3.4.2 Analysis

Every subgame in stage 1 is equivalent to the model in Section 3.3 after replacing $c(q)$ with $c(N)$ and removing q from the model. Hence, we can immediately characterize the social surplus (3.7) and the monopolist's profit under favorable expectations (3.9), unfavorable expectations (3.4), and potential maximization (3.13) for the subgames in stage 1.

In stage 0, the social planner and the monopolist choose the number of consumers N (eventually all of them become buyers) to maximize social surplus and profit respectively. Under C1 and B4, the solutions are uniquely characterized by the first-order conditions with respect to N of the respective functions. For expositional convenience, the following proposition characterizes the solutions by ignoring the integer constraint of N, replac-

[18] An equivalent timing is that the monopolist chooses both the price p and the number of consumers N in stage 1, and all consumers simultaneously decide whether to buy the good in stage 2.

ing summations $\sum_{i=1}^{N}$ with integrals $\int_0^N$, and assuming differentiability of the respective functions. In the appendix, I characterize the integer solutions of N.

Proposition 4 *By ignoring the integer constraint of N, replacing summations with integrals, and assuming $c(N)$, $v(i)$, $\alpha(i)$, and $\beta(i)$ are differentiable,*

(a) the socially optimal number of consumers N^{FB} is uniquely given by

$$c'(N^{FB}) = \underbrace{v(N^{FB})\alpha(N^{FB}) + \beta(N^{FB})}_{\textit{marginal consumer's benefit}} + \underbrace{\alpha'(N^{FB}) \int_0^{N^{FB}} v(i)di}_{\textit{marginal benefits to others}},$$

(b) the equilibrium number of consumers N^* under favorable expectations is uniquely given by

$$c'(N^*) = \underbrace{v(N^*)\alpha(N^*) + \beta(N^*)}_{\textit{marginal consumer's benefit}} + \alpha'(N^*) \underbrace{N^* v(N^*)}_{\textit{Spence distortion}} + \underbrace{N^*(\alpha(N^*)v'(N^*) + \beta'(N^*))}_{\textit{market power distortion}},$$

(c) the equilibrium number of consumers N^* under unfavorable expectations is uniquely given by

$$c'(N^*) = \beta(1),$$

(d) the equilibrium number of consumers N^* under potential maximization is uniquely given by

$$c'(N^*) = \underbrace{\frac{H(v|N^*)}{v(N^*)}}_{\textit{weight } (\geq 1)} \underbrace{(v(N^*)\alpha(N^*) + \beta(N^*))}_{\textit{marginal consumer's benefit}} + \underbrace{\left(1 - \frac{H(v|N^*)}{v(N^*)}\right)}_{\textit{weight } (\leq 0)} \underbrace{p^*(N^*)}_{\textit{price}}.$$[19]

As shown in Proposition 4, the socially optimal number of consumers equates the marginal cost with the marginal consumer's benefit plus the marginal network benefits to all other consumers. Compared to the social optimum, there are two distortions under favorable expectations. The first distortion is the classical *market power distortion*, in which the monopolist's marginal revenue falls short of its price. The second distortion

[19] $H(v|N) \equiv (\frac{1}{N}\sum_i \frac{1}{v(i)})^{-1}$ and $p^*(N) \equiv (\sum_i \frac{1}{v(i)})^{-1}(\sum_i \alpha(i) + \sum_i \frac{\beta(i)}{v(i)})$ are the respective counterparts of $H(v|q)$ in (3.12) and $p^*(q)$ in Proposition 2. By replacing summations with integrals, the former becomes $(\frac{1}{N}\int_i \frac{1}{v(i)}di)^{-1}$ and the latter becomes $(\int_i \frac{1}{v(i)}di)^{-1}(\int_i \alpha(i)di + \int_i \frac{\beta(i)}{v(i)}di)$.

is the Spence distortion, in which the monopolist internalizes the network benefit of the marginal instead of the average consumer. Under B3, both distortions put downward pressures on the equilibrium number of consumers. Hence, the monopolist attracts too few buyers relative to the social optimum. The equilibrium number of consumers under unfavorable expectations is uninteresting, which I will not discuss.

Under potential maximization, the monopolist extracts only part of the marginal consumer's surplus because $v(N^*)\alpha(N^*) + \beta(N^*) > p^*(N^*)$ (see footnote 16). Nevertheless, it extracts additional surplus from all other consumers due to the network benefits generated by the marginal consumer. Under B3 (which implies $H(v|N) \geq v(N)$), the sum of these two surpluses exceeds the marginal consumer's benefit. The monopolist may attract too many or too few buyers relative to the social optimum. The former occurs when the marginal network benefit $\alpha'(N^{FB})$ to other consumers (evaluated at the social optimum) is sufficiently small, whereas the latter occurs when (i) consumers do not differ too much in their network-benefit valuations (which implies $H(v|N) \approx v(N)$) and/or (ii) the marginal consumer's valuation is sufficiently low, i.e., $v(N^*)\alpha(N^*) + \beta(N^*) \approx p^*(N^*)$.

This section shows that all popular selection criteria (except unfavorable expectations) in the network economics literature give rise to the Spence distortion on the number of buyers/users, whereas potential maximization does not. Under favorable expectations, the monopolist attracts too few buyers relative to the social optimum due to both the market power and Spence distortions. By contrast, the monopolist may attract too many or too few buyers under potential maximization, and I have identified the key determinants.

3.5 Conclusion

This paper questions the existence of Spence distortions under strong network effects from both conceptual and theoretical perspectives. Under potential maximization, a refinement of Nash equilibrium justified by many solid microfoundations and experimental evidence, Spence distortions are absent in equilibrium. This finding has important policy implications for network markets. In these markets, the sources of market failure in quality (if any) might be those novel distortions identified in this paper but not Spence distortions. This suggests that regulations on network markets based on Spence distortion arguments might be socially suboptimal or even welfare-reducing.

Appendix A

Appendix for Chapter 1

Proof of Lemma 1 For all $\mathbf{p} \in P$, $i \in N$, and $\mathbf{x} \in X$,

$$\begin{aligned} u_i(\mathbf{x}) - p_i(x_i) &= w_i\Phi(\mathbf{x}) + \xi_i(\mathbf{x}_{-i}) - p_i(x_i) \quad \text{(by A1 and (1.2))} \\ &= w_i\Phi_{\mathbf{p}}(\mathbf{x}) + w_i\sum_{j\neq i}\frac{p_j(x_j)}{w_j} + \xi_i(\mathbf{x}_{-i}) \quad \text{(by (1.5))} \\ &= w_i\Phi_{\mathbf{p}}(\mathbf{x}) + \xi_i'(\mathbf{x}_{-i}). \quad (\xi_i'(\mathbf{x}_{-i}) \equiv w_i\sum_{j\neq i}\frac{p_j(x_j)}{w_j} + \xi_i(\mathbf{x}_{-i})) \end{aligned}$$

Proof of Lemma 2 It follows from the discussion in the text.

Proof of Lemma 3 It follows from the discussion in the text.

Proof of Proposition 1 The target action profile $\hat{\mathbf{x}}$ is fixed throughout the proof. For notational convenience, the optimal contracts (1.10) include all agents, but only those with $\hat{x}_i \neq o_i$ (i.e., belonging to $\hat{N}$) matter in the linear program (1.9); see also footnote 14. Without loss of generality, assume $\hat{N} = \{1, \ldots, |\hat{N}|\}$ and $w_1 \leq \cdots \leq w_{|\hat{N}|}$.

To prove this proposition, I first introduce some notation. Recall from p. 8 that $\bar{C}$ is symmetric. Therefore, agents can be partitioned into several groups so that for any two group members i and j, there exist mutual group members $k_1, \ldots, k_n \in \hat{N}$ ($n \geq 0$) such that $\hat{x}_j\bar{C}\hat{x}_{k_1}\bar{C}\hat{x}_{k_2}\bar{C}\cdots\bar{C}\hat{x}_{k_n}\bar{C}\hat{x}_i$.[1] Let $G_i \subseteq \hat{N}$ denote the set of agent i's group members together with agent i himself. Clearly, $G_i = G_j$ for all $j \in G_i$. A3 implies $\hat{x}_jC\hat{x}_i$ for all $j \in G_i\backslash\{i\}$. A2 implies $\hat{x}_jS\hat{x}_i$ for all $j \notin G_i$. For notational convenience, define $H_i \equiv \hat{N}\backslash G_i$,

[1] In graph theory terms, each vertex represents an agent, and agents i and j are linked iff $\hat{x}_j\bar{C}\hat{x}_i$. Every undirected graph can be decomposed into several connected components, which are the groups I have described.

$\tilde{\Phi} : 2^{\hat{N}} \to \mathbb{R}$ where $\tilde{\Phi}(Z) = \Phi(\mathbf{x})$ with $x_i = \hat{x}_i$ if $i \in Z$ ($Z \subseteq \hat{N}$) and $x_i = o_i$ otherwise, and $\tilde{u}_i : 2^{\hat{N}} \to \mathbb{R}$ is defined analogously. We can easily show that A1 and the above derived properties imply the following:

B1. for all $i \in \hat{N}$ and $Z \subseteq \hat{N} \backslash \{i\}$, $\tilde{u}_i(Z \cup \{i\}) - \tilde{u}_i(Z) = w_i[\tilde{\Phi}(Z \cup \{i\}) - \tilde{\Phi}(Z)]$, and

B2. for all $i \in \hat{N}$, $\tilde{\Phi}(G \cup H \cup \{i\}) - \tilde{\Phi}(G \cup H)$ is increasing in $G \subseteq G_i \backslash \{i\}$ and decreasing in $H \subseteq H_i$.

The linear program (1.9) is now re-expressed as

$$\max_{\hat{\mathbf{p}} \in \mathbb{R}^{|\hat{N}|}} \sum_{i \in \hat{N}} \hat{p}_i \quad \text{s.t.} \quad \sum_{i \in Z} \frac{\hat{p}_i}{w_i} \leq \tilde{\Phi}(\hat{N}) - \tilde{\Phi}(\hat{N} \backslash Z) \quad \text{for all } Z \subseteq \hat{N}. \tag{A.1}$$

The contracts (1.10) are re-expressed as

$$\hat{p}_i^* = \tilde{u}_i(\{1, \ldots, i\} \cup H_i) - \tilde{u}_i(\{1, \ldots, i-1\} \cup H_i) \quad \text{for all } i \in \hat{N}. \tag{A.2}$$

The rest of the linear programming proof consists of three steps. First, I show that $\hat{\mathbf{p}}^* \equiv (\hat{p}_i^*)_{i \in \hat{N}}$ is feasible. Next, I show that $\hat{\mathbf{p}}^*$ is optimal. Last, I show that $\hat{\mathbf{p}}^*$ is the unique optimal solution if $w_1 < \cdots < w_{|\hat{N}|}$.

Feasibility For $\hat{\mathbf{p}}^*$ to be feasible, we need to show

$$\sum_{i \in Z} \frac{\tilde{u}_i(\{1, \ldots, i\} \cup H_i) - \tilde{u}_i(\{1, \ldots, i-1\} \cup H_i)}{w_i} \leq \tilde{\Phi}(\hat{N}) - \tilde{\Phi}(\hat{N} \backslash Z) \quad \text{for all } Z \subseteq \hat{N}.$$

By B1, the above inequalities become

$$\sum_{i \in Z} [\tilde{\Phi}(\{1, \ldots, i\} \cup H_i) - \tilde{\Phi}(\{1, \ldots, i-1\} \cup H_i)] \leq \tilde{\Phi}(\hat{N}) - \tilde{\Phi}(\hat{N} \backslash Z) \quad \text{for all } Z \subseteq \hat{N}.$$

Note that any set $Z \subseteq \hat{N}$ takes the form of $\{i_1, \ldots, i_n\}$ where $i_1, \ldots, i_n \in \hat{N}$ and $i_1 < \cdots < i_n$ ($n \leq |\hat{N}|$). Thus, the respective inequality for the set $Z = \{i_1, \ldots, i_n\}$ is re-expressed as

$$\begin{aligned} & \sum_{m=1}^{n} [\tilde{\Phi}(\{1, \ldots, i_m\} \cup H_{i_m}) - \tilde{\Phi}(\{1, \ldots, i_m - 1\} \cup H_{i_m})] \\ \leq \ & \sum_{m=1}^{n} [\tilde{\Phi}(\hat{N} \backslash \{i_{m+1}, \ldots, i_n\}) - \tilde{\Phi}(\hat{N} \backslash \{i_m, \ldots, i_n\})]. \end{aligned}$$

For the above inequality to hold, it suffices to show that for all $m = 1, \ldots, n$,

$$\begin{aligned} & \tilde{\Phi}(\{1, \ldots, i_m - 1\} \cup H_{i_m} \cup \{i_m\}) - \tilde{\Phi}(\{1, \ldots, i_m - 1\} \cup H_{i_m}) \\ \leq \ & \tilde{\Phi}(\hat{N} \backslash \{i_m, \ldots, i_n\} \cup \{i_m\}) - \tilde{\Phi}(\hat{N} \backslash \{i_m, \ldots, i_n\}). \end{aligned}$$

Observe that $\{1, \ldots, i_m - 1\} \cup H_{i_m} = (G_{i_m} \cap \{1, \ldots, i_m - 1\}) \cup H_{i_m}$ and $\hat{N} \backslash \{i_m, \ldots, i_n\} = (G_{i_m} \backslash \{i_m, \ldots, i_n\}) \cup (H_{i_m} \backslash \{i_m, \ldots, i_n\})$. Hence, the above inequality holds by B2 because $G_{i_m} \cap \{1, \ldots, i_m - 1\} \subseteq G_{i_m} \backslash \{i_m, \ldots, i_n\}$ and $H_{i_m} \backslash \{i_m, \ldots, i_n\} \subseteq H_{i_m}$.

Optimality The proof of the optimality of $\hat{\mathbf{p}}^*$ consists of three steps. First, I derive the dual problem of (A.1). Next, I construct a feasible solution $\boldsymbol{\lambda}^*$ to the dual. Last, I show that the objective function value of the dual at $\boldsymbol{\lambda}^*$ is equal to that of the primal at $\hat{\mathbf{p}}^*$. The weak duality theorem states that the objective function value of the dual at any feasible solution is weakly greater than that of the primal at any feasible solution. This implies $\hat{\mathbf{p}}^*$ is optimal for the primal.

The dual problem is

$$\min_{(\lambda(Z) \geq 0)_{Z \subseteq \hat{N}}} \sum_{Z \subseteq \hat{N}} \lambda(Z) [\tilde{\Phi}(\hat{N}) - \tilde{\Phi}(\hat{N} \backslash Z)] \quad \text{s.t.} \quad \sum_{i \in Z \subseteq \hat{N}} \lambda(Z) = w_i \quad \text{for all } i \in \hat{N}.$$

Define $w_0 \equiv 0$, $i' \equiv \max\{j \in G_i \cup \{0\} | j < i\}$, and $\boldsymbol{\lambda}^* \equiv (\lambda^*(Z))_{Z \subseteq \hat{N}}$ where $\lambda^*(Z) = w_i - w_{i'}$ if $Z = G_i \cap \{i, i+1, \ldots, |\hat{N}|\}$ ($i \in \hat{N}$) and $\lambda^*(Z) = 0$ otherwise. The solution $\boldsymbol{\lambda}^*$ is feasible because for all $i \in \hat{N}$,

$$\begin{aligned}
\sum_{i \in Z \subseteq \hat{N}} \lambda^*(Z) &= \sum_{i \geq j \in G_i} \lambda^*(G_j \cap \{j, \ldots, |\hat{N}|\}) = \sum_{i \geq j \in G_i} (w_j - w_{j'}) \\
&= w_i - w_{i'} + \sum_{i' \geq j \in G_{i'}} (w_j - w_{j'}) \quad (G_i = G_{i'}) \\
&= w_i - w_{i'} + w_{i'} - w_{i''} + \cdots - w_0 = w_i - w_0 = w_i.
\end{aligned}$$

Denote $\bar{G} \equiv \bigcup_{i \in \hat{N}} \max G_i$. The objective function value of the dual at $\boldsymbol{\lambda}^*$ is

$$\begin{aligned}
& \sum_{Z \subseteq \hat{N}} \lambda^*(Z)[\tilde{\Phi}(\hat{N}) - \tilde{\Phi}(\hat{N} \backslash Z)] \\
= & \sum_{i \in \hat{N}} \lambda^*(G_i \cap \{i, \ldots, |\hat{N}|\})[\tilde{\Phi}(\hat{N}) - \tilde{\Phi}(\hat{N} \backslash (G_i \cap \{i, \ldots, |\hat{N}|\}))] \\
= & \sum_{j \in \bar{G}} \sum_{i \in G_j} (w_i - w_{i'})[\tilde{\Phi}(\hat{N}) - \tilde{\Phi}(\{1, \ldots, i-1\} \cup H_i)] \\
= & \tilde{\Phi}(\hat{N}) \sum_{j \in \bar{G}} \sum_{i \in G_j} (w_i - w_{i'}) - \sum_{j \in \bar{G}} \sum_{i \in G_j} w_i \tilde{\Phi}(\{1, \ldots, i-1\} \cup H_i) \\
+ & \sum_{j \in \bar{G}} \sum_{i \in G_j} w_{i'} \tilde{\Phi}(\{1, \ldots, i-1\} \cup H_i) \\
= & \tilde{\Phi}(\hat{N}) \sum_{i \in \bar{G}} w_i - \sum_{i \in \bar{G}} w_i \tilde{\Phi}(\{1, \ldots, i-1\} \cup H_i) - \sum_{j \in \bar{G}} \sum_{i \in G_j \backslash \bar{G}} w_i \tilde{\Phi}(\{1, \ldots, i-1\} \cup H_i) \\
+ & \sum_{j \in \bar{G}} \sum_{i \in G_j} w_{i'} \tilde{\Phi}(\{1, \ldots, i'\} \cup H_{i'}) \quad (\{i'+1, \ldots, i-1\} \subseteq H_i = H_{i'}) \\
= & \sum_{i \in \bar{G}} w_i[\tilde{\Phi}(\{1, \ldots, i\} \cup H_i) - \tilde{\Phi}(\{1, \ldots, i-1\} \cup H_i)] \quad (\{1, \ldots, i\} \cup H_i = \hat{N} \ \forall i \in \bar{G}) \\
+ & \sum_{j \in \bar{G}} \sum_{i \in G_j \backslash \bar{G}} w_i[\tilde{\Phi}(\{1, \ldots, i\} \cup H_i) - \tilde{\Phi}(\{1, \ldots, i-1\} \cup H_i)] \\
= & \sum_{j \in \bar{G}} \sum_{i \in G_j} w_i[\tilde{\Phi}(\{1, \ldots, i\} \cup H_i) - \tilde{\Phi}(\{1, \ldots, i-1\} \cup H_i)] \\
= & \sum_{i \in \hat{N}} [\tilde{u}_i(\{1, \ldots, i\} \cup H_i) - \tilde{u}_i(\{1, \ldots, i-1\} \cup H_i)] = \sum_{i \in \hat{N}} \hat{p}_i^*, \quad \text{(by B1 and (A.2))}
\end{aligned}$$

which is equal to the objective function value of the primal at $\hat{\mathbf{p}}^*$.

Uniqueness Denote $\tilde{p}_i \equiv \frac{\hat{p}_i}{w_i}$ and re-express (A.1) as

$$\max_{\tilde{\mathbf{p}} \in \mathbb{R}^{|\hat{N}|}} \sum_{i \in \hat{N}} w_i \tilde{p}_i \quad \text{s.t.} \quad \sum_{i \in Z} \tilde{p}_i \leq \tilde{\Phi}(\hat{N}) - \tilde{\Phi}(\hat{N} \backslash Z) \quad \text{for all } Z \subseteq \hat{N}. \tag{A.3}$$

For $\tilde{\mathbf{p}}^* \equiv \left(\frac{\hat{p}_i^*}{w_i}\right)_{i \in \hat{N}}$ (and thus $\hat{\mathbf{p}}^*$) to be the unique optimal solution, the necessary and sufficient condition provided by Mangasarian (1979, Theorem 1) is that $\tilde{\mathbf{p}}^*$ remains optimal for all linear programs obtained from (A.3) by an arbitrary but sufficiently small perturbation of the (cost) vector $(w_i)_{i \in \hat{N}}$. This condition is satisfied if $w_1 < \cdots < w_{|\hat{N}|}$ because any sufficiently small perturbation does not alter the ranking of w_i and, therefore, $\tilde{\mathbf{p}}^*$ remains optimal.

Proof of Corollary 1 It is a direct implication of Proposition 1.

Proof of Theorem 1 It remains to prove the "only if" part by contrapositive. Consider a two-agent game with $X_1 = X_2 = \{0, 1\}$ ($o_i = 0$), $u_1(\mathbf{x}) = u_2(\mathbf{x}) = 1$ if $\mathbf{x} = (1, 1)$ and $u_1(\mathbf{x}) = u_2(\mathbf{x}) = 0$ otherwise, and $U(\mathbf{x}, p_1(x_1) + p_2(x_2)) = V(\mathbf{x}) + p_1(x_1) + p_2(x_2)$ where $V(1, 0) = V(0, 1) = 1$ and $V(0, 0) = V(1, 1) = 0$. Clearly, u_1 and u_2 satisfy C1 (which implies A2 and A3); they also satisfy A1 with $w_1 = w_2 = 1$ and $\Phi = u_1 = u_2$. For notational convenience, denote $p_1 \equiv p_1(1)$ and $p_2 \equiv p_2(1)$.

There are exactly three types of contracts (p_1, p_2) leading to multiple equilibria in stage 2: (i) $p_i \leq 0$ and $p_j = 1$ ($i, j = 1, 2$; $i \neq j$), (ii) $p_i \geq 1$ and $p_j = 0$, and (iii) $(p_1, p_2) \in [0, 1] \times [0, 1]$. For the first type, there is a continuum of mixed-strategy equilibria in which $x_i = 1$ with probability 1 and $x_j = 1$ with any probability. Similarly, for the second type, there is a continuum of mixed-strategy equilibria in which $x_i = 0$ with probability 1 and $x_j = 1$ with any probability. For the third type, there are three equilibria: (i) $\mathbf{x} = (1, 1)$, (ii) $\mathbf{x} = (0, 0)$, and (iii) the mixed-strategy equilibrium in which $x_i = 1$ with probability p_j.

For the first (second) type, all equilibria have the same potential of $-p_i$ (0). Recall from p. 11 that the principal can select among potential maximizers under potential maximization. Therefore, she can always select the equilibrium giving her the highest payoff for both types.[2] For the third type, we can easily show that the principal's expected payoffs in those three equilibria are $p_1 + p_2$, 0, and $p_1 + p_2$ respectively, and the potential maximizer is $\mathbf{x} = (1, 1)$ if $p_1 + p_2 \leq 1$ and $\mathbf{x} = (0, 0)$ if $p_1 + p_2 \geq 1$. Observe that potential maximization selects the equilibrium giving the principal the highest payoff if and only if $p_1 + p_2 \leq 1$.

If the underlying equilibrium selection criterion is not more pessimistic than potential maximization, there exists a non-empty subset of the third type of contracts $\mathcal{P} \subseteq \{(p_1, p_2) \in (0, 1] \times (0, 1] | p_1 + p_2 > 1\}$ in which either $\mathbf{x} = (1, 1)$ or the mixed-strategy equilibrium is selected; both give the principal the same expected payoff of $p_1 + p_2 > 1$. If instead the principal offers the $\mathbf{w}$-DC contracts, the optimal action profiles are $(1, 1)$, $(1, 0)$, and $(0, 1)$ by Corollary 2; in either case, her payoff is only 1.

Proof of Corollary 2 It follows from the discussion in the text.

Proof of Corollary 3 It follows from the discussion in the text.

Proof of Lemma 4 First, note that

$$\sum_j \sum_{k \in E_j} \theta_j \theta_k x_j x_k = \sum_j \left(\theta_j x_j \sum_{k \in E_j} \theta_k x_k \right) = \theta_i x_i \sum_{k \in E_i} \theta_k x_k + \sum_{j \neq i} \left(\theta_j x_j \sum_{k \in E_j} \theta_k x_k \right)$$

$$= \theta_i x_i \sum_{k \in E_i} \theta_k x_k + \sum_{j \neq i} \left(\theta_j x_j \theta_i x_i \cdot 1_{i \in E_j} + \theta_j x_j \sum_{k \in E_j \setminus \{i\}} \theta_k x_k \right)$$

$$= \theta_i x_i \sum_{k \in E_i} \theta_k x_k + \theta_i x_i \sum_{j \in E_i} \theta_j x_j + \sum_{j \neq i} \left(\theta_j x_j \sum_{k \in E_j \setminus \{i\}} \theta_k x_k \right) \quad (i \in E_j \text{ iff } j \in E_i)$$

$$= 2\theta_i x_i \sum_{j \in E_i} \theta_j x_j + \sum_{j \neq i} \sum_{k \in E_j \setminus \{i\}} \theta_j \theta_k x_j x_k.$$

[2] For the first type, all equilibria give her the same expected payoff of $1 + p_i$. For the second type, the pure-strategy equilibrium in which $x_i = 0$ and $x_j = 1$ gives her the highest payoff of 1.

For all $i \in N$ and $\mathbf{x} \in X$,

$$\begin{aligned}
w_i\Phi(\mathbf{x}) &= \frac{v_i}{\theta_i}\left(\sum_j \frac{\theta_j b_j(x_j)}{v_j} + \frac{1}{2}\sum_j \sum_{k\in E_j} \theta_j\theta_k x_j x_k\right) \quad \text{(by (1.12))} \\
&= b_i(x_i) + \frac{v_i}{\theta_i}\sum_{j\neq i}\frac{\theta_j b_j(x_j)}{v_j} + v_i x_i \sum_{j\in E_i}\theta_j x_j + \frac{v_i}{2\theta_i}\sum_{j\neq i}\sum_{k\in E_j\setminus\{i\}}\theta_j\theta_k x_j x_k \\
&= u_i(\mathbf{x}) - \xi_i(\mathbf{x}_{-i}). \quad \text{(by (1.3); } \xi_i(\mathbf{x}_{-i}) \equiv -\frac{v_i}{\theta_i}\sum_{j\neq i}\frac{\theta_j b_j(x_j)}{v_j} - \frac{v_i}{2\theta_i}\sum_{j\neq i}\sum_{k\in E_j\setminus\{i\}}\theta_j\theta_k x_j x_k)
\end{aligned}$$

Proof of Corollary 4 It is a direct application of Theorem 1, Corollary 1, and Lemma 4.

Proof of Lemma 5 For all $i \in N$ and $\mathbf{x} \in X$,

$$\begin{aligned}
w_i\Phi(\mathbf{x}) &= v_i\left(\sum_j \frac{b_j(x_j)}{v_j} + g(\mathbf{x})\right) \quad \text{(by (1.15))} \\
&= b_i(x_i) + v_i\sum_{j\neq i}\frac{b_j(x_j)}{v_j} + v_i g(\mathbf{x}) \\
&= u_i(\mathbf{x}) - \xi_i(\mathbf{x}_{-i}). \quad \text{(by (1.14) and } \xi_i(\mathbf{x}_{-i}) \equiv -v_i\sum_{j\neq i}\frac{b_j(x_j)}{v_j})
\end{aligned}$$

Proof of Corollary 5 It is a direct application of Theorem 1 and Lemma 5.

Proof of Lemma 6 For all $i \in N$ and $\mathbf{x} \in X$,

$$\begin{aligned}
w_i\Phi(\mathbf{x}) &= b_i(x_i) + \frac{1}{2}\delta v_i\theta_i x_i^2 + w_i\sum_{j\neq i}\frac{b_j(x_j) + \frac{1}{2}\delta v_j\theta_j x_j^2}{w_j} + \frac{w_i}{2}(\sum_j \theta_j x_j)^2 \quad \text{(by (1.16))} \\
&= b_i(x_i) + \frac{1}{2}\delta v_i\theta_i x_i^2 + w_i\sum_{j\neq i}\frac{b_j(x_j) + \frac{1}{2}\delta v_j\theta_j x_j^2}{w_j} \\
&+ \frac{\delta v_i}{2\theta_i}(\theta_i x_i + \sum_{j\neq i}\theta_j x_j)^2 + \frac{(1-\delta)v_i}{2}(\sum_j \theta_j x_j)^2 \\
&= b_i(x_i) + \frac{1}{2}\delta v_i\theta_i x_i^2 + w_i\sum_{j\neq i}\frac{b_j(x_j) + \frac{1}{2}\delta v_j\theta_j x_j^2}{w_j} \\
&+ \frac{1}{2}\delta v_i\theta_i x_i^2 + \delta v_i x_i\sum_{j\neq i}\theta_j x_j + \frac{\delta v_i}{2\theta_i}(\sum_{j\neq i}\theta_j x_j)^2 + \frac{(1-\delta)v_i}{2}(\sum_j \theta_j x_j)^2
\end{aligned}$$

$$
\begin{aligned}
&= \quad u_i(\mathbf{x}) - \xi_i'(\mathbf{x}_{-i}), \quad \text{where} \\
&\quad \xi_i'(\mathbf{x}_{-i}) \equiv (1-\delta)\xi_i(\mathbf{x}_{-i}) - w_i \sum_{j\neq i} \frac{b_j(x_j) + \frac{1}{2}\delta v_j \theta_j x_j^2}{w_j} - \frac{\delta v_i}{2\theta_i}(\sum_{j\neq i} \theta_j x_j)^2.
\end{aligned}
$$

Proof of Corollary 6 It is a direct application of Theorem 1, Corollary 1, and Lemma 6.

Generic Uniqueness of the Weighted Potential Maximizer

Suppose a game $\mathcal{G} \equiv (N, X, \mathbf{u})$ is a weighted potential game. Given a potential function Φ (together with a weight vector $\mathbf{w} \in \mathbb{R}^N_{++}$) of $\mathcal{G}$, it is clear that the maximizer of Φ is generically unique. However, it is unclear whether another potential function Φ' (together with another weight vector $\mathbf{w}' \in \mathbb{R}^N_{++}$) of $\mathcal{G}$ has the same maximizer(s). Therefore, the exact statement to prove is as follows. To the best of my knowledge, this paper is the first to give a direct proof of this statement.

Lemma 0 *The set of potential maximizers of a weighted potential game is independent of the choice of the potential function.*

Proof. Suppose $(\mathbf{w}, \Phi)$ and $(\mathbf{w}', \Phi')$ are two choices of "weight-potential" pairs. By the definition of weighted potential games (A1), for all $i \in N$, $x_i, x_i' \in X_i$, and $\mathbf{x}_{-i} \in X_{-i}$,

$$u_i(x_i, \mathbf{x}_{-i}) - u_i(x_i', \mathbf{x}_{-i}) = w_i[\Phi(x_i, \mathbf{x}_{-i}) - \Phi(x_i', \mathbf{x}_{-i})] = w_i'[\Phi'(x_i, \mathbf{x}_{-i}) - \Phi'(x_i', \mathbf{x}_{-i})]. \quad \text{(A.4)}$$

Denote $\tilde{w}_i \equiv \frac{w_i'}{w_i}$. Clearly, $\tilde{w}_i > 0$ for all i. Without loss of generality, assume $\tilde{w}_1 \leq \cdots \leq \tilde{w}_N$. It remains to show that $\bar{\mathbf{x}} \in \arg\max_{\mathbf{x}\in X} \Phi'(\mathbf{x})$ implies $\bar{\mathbf{x}} \in \arg\max_{\mathbf{x}\in X} \Phi(\mathbf{x})$. To see this, for each $\mathbf{x} \in X$,

$$\begin{aligned}
& \Phi(\bar{\mathbf{x}}) - \Phi(\mathbf{x}) \\
= \;& \sum_{i=1}^{N} [\Phi(\bar{x}_1, \ldots, \bar{x}_i, x_{i+1}, \ldots, x_N) - \Phi(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \ldots, x_N)] \\
= \;& \sum_{i=1}^{N} \tilde{w}_i [\Phi'(\bar{x}_1, \ldots, \bar{x}_i, x_{i+1}, \ldots, x_N) - \Phi'(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \ldots, x_N)] \quad \text{(by (A.4))} \\
\geq \;& \sum_{i=1}^{N-1} \tilde{w}_i [\Phi'(\bar{x}_1, \ldots, \bar{x}_i, x_{i+1}, \ldots, x_N) - \Phi'(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \ldots, x_N)] \\
+ \;& \tilde{w}_{N-1} [\Phi'(\bar{\mathbf{x}}) - \Phi'(\bar{x}_1, \ldots, \bar{x}_{N-1}, x_N)] \\
= \;& \sum_{i=1}^{N-2} \tilde{w}_i [\Phi'(\bar{x}_1, \ldots, \bar{x}_i, x_{i+1}, \ldots, x_N) - \Phi'(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \ldots, x_N)] \\
+ \;& \tilde{w}_{N-1} [\Phi'(\bar{\mathbf{x}}) - \Phi'(\bar{x}_1, \ldots, \bar{x}_{N-2}, x_{N-1}, x_N)] \\
\geq \;& \sum_{i=1}^{N-2} \tilde{w}_i [\Phi'(\bar{x}_1, \ldots, \bar{x}_i, x_{i+1}, \ldots, x_N) - \Phi'(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \ldots, x_N)] \\
+ \;& \tilde{w}_{N-2} [\Phi'(\bar{\mathbf{x}}) - \Phi'(\bar{x}_1, \ldots, \bar{x}_{N-2}, x_{N-1}, x_N)] \\
\geq \;& \cdots \geq \tilde{w}_1 [\Phi'(\bar{\mathbf{x}}) - \Phi'(\mathbf{x})] \geq 0. \quad (\bar{\mathbf{x}} \in \arg\max_{\mathbf{x} \in X} \Phi'(\mathbf{x})) \quad \blacksquare
\end{aligned}$$

Appendix B

Appendix for Chapter 2

Alternative Pricing Instruments

For the five examples mentioned on p. 26, nightclubs, shopping malls, and open access journals charge subscription fees, whereas credit cards and daily deal sites charge transaction fees. I now modify the model in Section 2.3 to study the latter.

Model If a side-i agent joins platform m, he now pays a transaction fee $p_i^m \in \mathbb{R}$ per each interaction with side-j agents who the same platform. Thus, his respective payoffs from joining A and B are

$$u_i^A(n_j, p_i^A) = (v_i^A - p_i^A)n_j, \quad u_i^B(n_j, p_i^B) = (v_i^B - p_i^B)(N_j - n_j).^1 \tag{B.1}$$

The platforms' profits are the total transaction fees collected from both sides:

$$\pi^A(n_1, n_2, p_1^A, p_2^A) = (p_1^A + p_2^A)n_1 n_2, \quad \pi^B(n_1, n_2, p_1^B, p_2^B) = (p_1^B + p_2^B)(N_1 - n_1)(N_2 - n_2).$$

I assume neither platform uses weakly dominated strategies, i.e., the sum of transaction fees on both sides $p_1^m + p_2^m$ is non-negative. The rest of the model setup is the same as that of the subscription model.

Analysis There are two equilibria in stage 2 when $p_i^m \leq v_i^m$ for all i, m:[2] (i) all agents join A and (ii) all agents join B. Both Pareto dominance and Pareto-dominated selection

[1] Note that there is a one-to-one mapping between transaction fees and revenue sharing, in which platform A (similarly for B) charges side i a rate of $r_i^A \in \mathbb{R}$ and $u_i^A(n_j, r_i^A) = (1 - r_i^A)v_i^A n_j$.

[2] Clearly, neither platform will set a transaction fee p_i^m above the respective per-interaction benefit v_i^m in equilibrium.

are inapplicable for the same reason as before, whereas potential maximization remains applicable. Observe that agents' payoffs (B.1) in the current model can be obtained by replacing v_i^m and p_i^m in (2.5) with $v_i^m - p_i^m$ and 0 respectively. Thus, every subgame of this model is a weighted potential game, and we can immediately characterize the potential maximizer by applying the above replacements to Lemma 2.

Lemma 3 *When $p_i^m \leq v_i^m$ for all $i = 1, 2$ and $m = A, B$, the potential maximizer is all agents joining platform A if $(v_1^A - p_1^A)(v_2^A - p_2^A) \geq (v_1^B - p_1^B)(v_2^B - p_2^B)$ and all agents joining platform B otherwise.*

As shown in Lemma 3, under potential maximization, all agents coordinate on the platform with the higher value of $(v_1^m - p_1^m)(v_2^m - p_2^m)$. Hence, stage 1 is analogous to standard Bertrand competition as in the subscription model. Generically and w.l.o.g., assume A (B) is the dominant (dominated) platform in equilibrium. The standard analysis of Bertrand competition implies that B sets the minimum prices to maximize $(v_1^B - p_1^B)(v_2^B - p_2^B)$ and A slightly undercuts B to capture the entire market. Under the non-negative profit constraint $p_1^B + p_2^B \geq 0$, B's equilibrium prices are $p_1^{B*} = \frac{1}{2}(v_1^B - v_2^B)$ and $p_2^{B*} = \frac{1}{2}(v_2^B - v_1^B)$. Hence, from Lemma 3, A's equilibrium prices satisfy

$$(v_1^A - p_1^{A*})(v_2^A - p_2^{A*}) = \left(v_1^B - \frac{v_1^B - v_2^B}{2}\right)\left(v_2^B - \frac{v_2^B - v_1^B}{2}\right) = \left(\frac{v_1^B + v_2^B}{2}\right)^2. \tag{B.2}$$

Under the constraint $p_1^A + p_2^A \geq 0$, it is easy to see that A can successfully undercut B if and only if $v_1^A + v_2^A > v_1^B + v_2^B$. Suppose indeed $v_1^A + v_2^A > v_1^B + v_2^B$. Thus, A maximizes its profit by optimally allocating the prices on the two sides subject to (B.2), i.e.,

$$\max_{p_i^A \leq v_i^A} (p_1^A + p_2^A) N_1 N_2 \quad \text{s.t.} \quad (v_1^A - p_1^A)(v_2^A - p_2^A) = \left(\frac{v_1^B + v_2^B}{2}\right)^2.$$

Solving the above problem gives us platform A's optimal pricing strategy.

Proposition 4 *Suppose $v_1^A + v_2^A > v_1^B + v_2^B$. Under potential maximization, stage 1 is a Bertrand equilibrium with*

$$p_1^{A*} = v_1^A - \frac{v_1^B + v_2^B}{2}, \quad p_2^{A*} = v_2^A - \frac{v_1^B + v_2^B}{2}, \quad p_1^{B*} = \frac{v_1^B - v_2^B}{2}, \quad p_2^{B*} = \frac{v_2^B - v_1^B}{2}.$$

All agents join platform A in stage 2, and A's equilibrium profit is given by $\pi^{A*} = \left(v_1^A + v_2^A - v_1^B - v_2^B\right) N_1 N_2$.

Discussion In contrast to the subscription model, the market now tips to the platform with the higher sum of per-interaction benefits $v_1^m + v_2^m$ instead of the product of them $v_1^m v_2^m$. Therefore, for these two models, the dominant platform may differ even with the same set of parameter values. Take (2.8) as an example: the dominant platform is A in the subscription model but B in this model. Following Section 2.3.3, I discuss the three key implications under the current framework.

Divide and Conquer As shown in Proposition 4, the dominated platform (B) rebalances the net per-interaction benefits $v_i^B - p_i^B$ of the two sides by charging the side with the higher per-interaction benefit and subsidizing the other. The resulting net per-interaction benefit is $\frac{1}{2}(v_1^B + v_2^B)$ on both sides. Similarly, the dominant platform (A) adjusts the net per-interaction benefits $v_i^A - p_i^A$ of the two sides so that the resulting net per-interaction benefit is also $\frac{1}{2}(v_1^B + v_2^B)$ on both sides. To achieve this, A needs to divide and conquer if its competitor is sufficiently strong (in terms of the value of $v_1^B + v_2^B$); otherwise, A can charge both sides.[3]

Divide/Conquer Side The divide/conquer side of the dominated platform (B) depends only on the relative size of its own per-interaction benefits v_1^B and v_2^B. As explained, the dominant platform (A) monetizes both sides if B is weak. But whenever A divides and conquers, observe from Proposition 4 that the divide/conquer side depends only on the relative size of v_1^A and v_2^A. This differs from that of the subscription model in which the divide/conquer side of A also depends on v_1^B and v_2^B.

[3] When $v_1^B = v_2^B = 0$, B's equilibrium prices are $p_1^{B*} = p_2^{B*} = 0$ by Proposition 4. It is as if A is a monopolist and B is the outside option of not joining the platform. In this case, A sets $p_1^{A*} = v_1^A$ and $p_2^{A*} = v_2^A$ and extracts all agents' surplus.

Optimal Design Given that platforms can rebalance the net per-interaction benefits of the two sides with transaction fees, the optimal design of both platforms is to maximize the sum of per-interaction benefits $v_1^m + v_2^m$. Unlike the subscription model, all agents now always coordinate on the platform delivering the higher social surplus $(v_1^m + v_2^m)N_1N_2$. Clearly, the optimal design of platforms now also maximizes social surplus.

Two-Part Tariffs The analysis is readily extended to two-part tariffs. Suppose the respective payoffs of a side-i agent from joining A and B are

$$u_i^A(n_j, p_i^A, q_i^A) = (v_i^A - p_i^A)n_j - q_i^A, \quad u_i^B(n_j, p_i^B, q_i^B) = (v_i^B - p_i^B)(N_j - n_j) - q_i^B,$$

where p_i^m and q_i^m are respectively the transaction and subscription fees set by platform m to side i. Hence, A's profit is

$$\pi^A(n_1, n_2, p_1^A, p_2^A, q_1^A, q_2^A) = (p_1^A + p_2^A)n_1n_2 + q_1^A n_1 + q_2^A n_2,$$

and similarly for B. Analogous to the transaction model, agents' payoffs in this model can be obtained by replacing v_i^m and p_i^m in (2.5) with $v_i^m - p_i^m$ and q_i^m respectively. Thus, every subgame of the current model is a weighted potential game, and we can immediately characterize the potential maximizer.

The following analysis assumes that (i) subscription fees q_i^m are non-negative as in the subscription model and (ii) platforms do not use weakly dominated strategies as in the transaction model. The latter implies $p_1^m + p_2^m \geq 0$ if $q_1^m = q_2^m = 0$. As I will prove shortly, the equilibrium under potential maximization is the same as that of the transaction model, i.e., characterized by Proposition 4. In other words, when both transaction and subscription fees are available, only the former are used. Intuitively, transaction fees alleviate the cost of miscoordination and, therefore, are superior to subscription fees.

Proof. I directly verify that the equilibrium is given by Proposition 4. Assume as in Proposition 4 that $v_1^A + v_2^A > 2\bar{v}$ where $\bar{v} \equiv \frac{1}{2}(v_1^B + v_2^B)$. Fixing B's equilibrium prices $(p_1^{B*}, p_2^{B*}, q_1^{B*}, q_2^{B*}) = (v_1^B - \bar{v}, v_2^B - \bar{v}, 0, 0)$ and applying the corresponding replacements to (2.7), A's profit maximization problem is

$$\max_{p_i^A \le v_i^A, q_i^A \ge 0} (p_1^A + p_2^A) N_1 N_2 + q_1^A N_1 + q_2^A N_2 \quad \text{s.t.} \tag{B.3}$$

$$\frac{v_2^A - p_2^A + \bar{v}}{N_2} q_1^A + \frac{v_1^A - p_1^A + \bar{v}}{N_1} q_2^A = (v_1^A - p_1^A)(v_2^A - p_2^A) - \bar{v}^2.$$

I now solve the problem and show that A's optimal prices are indeed given by Proposition 4. Observe from the constraint that $q_1^A, q_2^A \ge 0$ implies $(v_1^A - p_1^A)(v_2^A - p_2^A) \ge \bar{v}^2$. First, fixing (p_1^A, p_2^A), I derive the optimal (q_1^A, q_2^A). W.l.o.g., assume $v_1^A - p_1^A \le v_2^A - p_2^A$. By Proposition 3 (with the corresponding replacements), we have

$$q_1^{A*} = 0, \quad q_2^{A*} = \frac{(v_1^A - p_1^A)(v_2^A - p_2^A) - \bar{v}^2}{v_1^A - p_1^A + \bar{v}} N_1.$$

Hence, A's problem (B.3) becomes

$$\max_{p_i^A \le v_i^A, (v_1^A - p_1^A)(v_2^A - p_2^A) \ge \bar{v}^2, v_1^A - p_1^A \le v_2^A - p_2^A} (p_1^A + p_2^A) N_1 N_2 + \frac{(v_1^A - p_1^A)(v_2^A - p_2^A) - \bar{v}^2}{v_1^A - p_1^A + \bar{v}} N_1 N_2, \tag{B.4}$$

which is simplified to

$$\max_{p_i^A \le v_i^A, (v_1^A - p_1^A)(v_2^A - p_2^A) \ge \bar{v}^2, v_1^A - p_1^A \le v_2^A - p_2^A} \frac{(v_1^A - p_1^A + \bar{v} - v_2^A) p_1^A + \bar{v} p_2^A + v_1^A v_2^A - \bar{v}^2}{v_1^A - p_1^A + \bar{v}} N_1 N_2.$$

Observe that the profit above is increasing in p_2^A. Therefore, A optimally sets the highest possible p_2^A, so that $v_1^A - p_1^A = v_2^A - p_2^A \equiv z$. Thus, A's problem (B.4) becomes

$$\max_{z \ge \bar{v}} (v_1^A - z + v_2^A - z) N_1 N_2 + \frac{z^2 - \bar{v}^2}{z + \bar{v}} N_1 N_2,$$

which is simplified to

$$\max_{z \ge \bar{v}} (v_1^A + v_2^A - z - \bar{v}) N_1 N_2.$$

Clearly, A optimally sets $z^* = \bar{v}$, and thus $(p_1^{A*}, p_2^{A*}, q_1^{A*}, q_2^{A*}) = (v_1^A - \bar{v}, v_2^A - \bar{v}, 0, 0)$ as in Proposition 4. It is easy to see that B has no profitable deviation when A sets the above equilibrium prices. ■

Heterogeneous Agents, Multiple Platforms, Price Discrimination

In this appendix, agents are heterogeneous and may choose among more than two actions. It is helpful to first introduce some notation. Let S, S_1, and S_2 denote the set of all agents, side-1 agents, and side-2 agents respectively. Clearly, $S_1 \cup S_2 = S$, $S_1 \cap S_2 = \emptyset$, and $|S_i| = N_i$. Denote $x_s \in X_s$ as the action of agent $s \in S$, $\mathbf{x} \in X \equiv \prod_s X_s$ as the action profile, and $\mathbf{x}_{-s} \in X_{-s} \equiv \prod_{t \neq s} X_t$ as the action profile except x_s. Let $u_s(\mathbf{x}, \mathbf{p})$ denote the payoff of agent s with prices $\mathbf{p}$ set by platforms in stage 1.

Using the above notation, the model in Section 2.3 is characterized by $X_s = \{A, B\}$ for all s and the payoff of agent s from side $i = 1, 2$ (i.e., $s \in S_i$) given by

$$u_s(A, \mathbf{x}_{-s}, \mathbf{p}) = v_i^A |\{t \in S_j : x_t = A\}| - p_i^A, \quad u_s(B, \mathbf{x}_{-s}, \mathbf{p}) = v_i^B |\{t \in S_j : x_t = B\}| - p_i^B.$$

Before studying the generalizations, I first give the general definition of a weighted potential game for a subgame in stage 2.

Definition 1 A subgame with $\mathbf{p}$ set by platforms in stage 1 is a *weighted potential game* if there exists a (weight) vector $(w_s(\mathbf{p}))_s \in \mathbb{R}_+^{|S|}$ and a (potential) function $\Phi_{\mathbf{p}} : X \to \mathbb{R}$ such that for all $s \in S$ and $\mathbf{x}_{-s} \in X_{-s}$,

$$u_s(x_s, \mathbf{x}_{-s}, \mathbf{p}) - u_s(x_s', \mathbf{x}_{-s}, \mathbf{p}) = w_s(\mathbf{p})[\Phi_{\mathbf{p}}(x_s, \mathbf{x}_{-s}) - \Phi_{\mathbf{p}}(x_s', \mathbf{x}_{-s})] \quad \text{for all } x_s, x_s' \in X_s.$$

Now, consider the first generalization of the model in Section 2.3 where the payoff of agent $s \in S_i$ is

$$u_s(A, \mathbf{x}_{-s}, \mathbf{p}) = v_s^A \sum_{t \in S_j : x_t = A} \theta_t + d_s^A - p_i^A, \quad u_s(B, \mathbf{x}_{-s}, \mathbf{p}) = v_s^B \sum_{t \in S_j : x_t = B} \theta_t + d_s^B - p_i^B,$$

where $\theta_t \in \mathbb{R}_{++}$ measures the importance of agent t to agents on the other side, and $d_s^m \in \mathbb{R}$ is the stand-alone benefit/cost of joining platform m for agent s. If all agents have the same importance θ_s (which can then be normalized to one) as in the previous model, then they only care about the number (but not the identity) of participants on the other side.

I verify that every subgame is a weighted potential game with $w_s(\mathbf{p}) = \frac{v_s^A + v_s^B}{\theta_s}$ and

$$\begin{aligned}\Phi_{\mathbf{p}}(\mathbf{x}) &= \sum_{s\in S_1:x_s=A}\theta_s \sum_{s\in S_2:x_s=A}\theta_s \\ &\quad - \sum_{s\in S_1:x_s=A}\frac{\theta_s(v_s^B\sum_{t\in S_2}\theta_t - d_s^A + d_s^B + \tilde{p}_1)}{v_s^A+v_s^B} \\ &\quad - \sum_{s\in S_2:x_s=A}\frac{\theta_s(v_s^B\sum_{t\in S_1}\theta_t - d_s^A + d_s^B + \tilde{p}_2)}{v_s^A+v_s^B}.\end{aligned}$$

Proof. Consider agent $s \in S_i$,

$$\begin{aligned}& w_s(\mathbf{p})[\Phi_{\mathbf{p}}(A,\mathbf{x}_{-s}) - \Phi_{\mathbf{p}}(B,\mathbf{x}_{-s})] \\ =\ & \frac{v_s^A+v_s^B}{\theta_s}\left[\theta_s\sum_{t\in S_j:x_t=A}\theta_t - \frac{\theta_s(v_s^B\sum_{t\in S_j}\theta_t - d_s^A + d_s^B + \tilde{p}_i)}{v_s^A+v_s^B}\right] \\ =\ & v_s^A\sum_{t\in S_j:x_t=A}\theta_t + d_s^A - p_i^A - v_s^B(\sum_{t\in S_j}\theta_t - \sum_{t\in S_j:x_t=A}\theta_t) - d_s^B + p_i^B \\ =\ & u_s(A,\mathbf{x}_{-s},\mathbf{p}) - u_s(B,\mathbf{x}_{-s},\mathbf{p}). \quad \blacksquare\end{aligned}$$

If each agent derives the same per-interaction benefit v_s at either platform, then we can allow each of them to have different importance θ_s^m at different platforms. Analogously, we can verify that every subgame is a weighted potential game with $w_s(\mathbf{p}) = \frac{v_s}{\theta_s^A+\theta_s^B}$ and

$$\begin{aligned}\Phi_{\mathbf{p}}(\mathbf{x}) &= \sum_{s\in S_1:x_s=A}(\theta_s^A+\theta_s^B) \sum_{s\in S_2:x_s=A}(\theta_s^A+\theta_s^B) \\ &\quad - \sum_{s\in S_1:x_s=A}\frac{(\theta_s^A+\theta_s^B)(v_s\sum_{t\in S_2}\theta_t^B - d_s^A + d_s^B + \tilde{p}_1)}{v_s} \\ &\quad - \sum_{s\in S_2:x_s=A}\frac{(\theta_s^A+\theta_s^B)(v_s\sum_{t\in S_1}\theta_t^B - d_s^A + d_s^B + \tilde{p}_2)}{v_s}.\end{aligned}$$

Alternatively, if each side-1 agent has the same importance θ_s and per-interaction benefit v_s at either platform, then we can allow each side-2 agent to have different importance θ_s^m and per-interaction benefits v_s^m at different platforms. We can verify that every subgame is a weighted potential game with $w_s(\mathbf{p}) = \frac{v_s}{\theta_s}$ for all $s \in S_1$, $w_s(\mathbf{p}) = \frac{v_s^A+v_s^B}{\theta_s^A+\theta_s^B}$ for all

$s \in S_2$, and

$$\begin{aligned}\Phi_{\mathbf{p}}(\mathbf{x}) &= \sum_{s\in S_1:x_s=A}\theta_s\sum_{s\in S_2:x_s=A}(\theta_s^A+\theta_s^B)-\sum_{s\in S_1:x_s=A}\frac{\theta_s(v_s\sum_{t\in S_2}\theta_t^B-d_s^A+d_s^B+\tilde{p}_1)}{v_s}\\&\quad-\sum_{s\in S_2:x_s=A}\frac{(\theta_s^A+\theta_s^B)(v_s^B\sum_{t\in S_1}\theta_t-d_s^A+d_s^B+\tilde{p}_2)}{v_s^A+v_s^B}.\end{aligned}$$

I now further extend the model to M competing platforms given that each agent has the same importance θ_s and per-interaction benefit v_s at any platform.[4] All platforms simultaneously set prices $(p_1^m, p_2^m) \in \mathbb{R}^2$ in stage 1 ($m = 1, \ldots, M$), and all agents simultaneously decide which platform (if any) to join in stage 2. Each agent's action set is now $X_s = \{0 \equiv \text{not join}, 1 \equiv \text{join } 1, \ldots, M \equiv \text{join } M\}$. The payoff of agent $s \in S_i$ is

$$u_s(m,\mathbf{x}_{-s},\mathbf{p}) = v_s\sum_{t\in S_j:x_t=m}\theta_t+d_s^m-p_i^m,\quad u_s(0,\mathbf{x}_{-s},\mathbf{p})=0.$$

I verify that every subgame is a weighted potential game with $w_s(\mathbf{p}) = \frac{v_s}{\theta_s}$ and

$$\begin{aligned}\Phi_{\mathbf{p}}(\mathbf{x}) &= \sum_{m=1}^{M}(\sum_{s\in S_1:x_s=m}\theta_s\sum_{s\in S_2:x_s=m}\theta_s)-\sum_{s\in S_1:x_s=m\neq 0}\frac{\theta_s(p_1^m-d_s^m)}{v_s}\\&\quad-\sum_{s\in S_2:x_s=m\neq 0}\frac{\theta_s(p_2^m-d_s^m)}{v_s}.\end{aligned}$$

Proof. Consider agent $s \in S_i$. When $m, m' \in \{1, \ldots, M\}$,

$$\begin{aligned}&w_s(\mathbf{p})[\Phi_{\mathbf{p}}(m,\mathbf{x}_{-s})-\Phi_{\mathbf{p}}(m',\mathbf{x}_{-s})]\\&=\frac{v_s}{\theta_s}\left[\theta_s\sum_{t\in S_j:x_t=m}\theta_t-\frac{\theta_s(p_i^m-d_s^m)}{v_s}-\theta_s\sum_{t\in S_j:x_t=m'}\theta_t+\frac{\theta_s(p_i^{m'}-d_s^{m'})}{v_s}\right]\\&= v_s\sum_{t\in S_j:x_t=m}\theta_t+d_s^m-p_i^m-v_s\sum_{t\in S_j:x_t=m'}\theta_t-d_s^{m'}+p_i^{m'}\\&= u_s(m,\mathbf{x}_{-s},\mathbf{p})-u_s(m',\mathbf{x}_{-s},\mathbf{p}).\end{aligned}$$

[4] See Tan and Zhou (forthcoming) for a similar model, which also allows for any number of platforms. They ensure a unique equilibrium by assuming sufficiently small network effects relative to the heterogeneity of agents.

When $m \in \{1, \ldots, M\}$ and $m' = 0$,

$$w_s(\mathbf{p})[\Phi_{\mathbf{p}}(m, \mathbf{x}_{-s}) - \Phi_{\mathbf{p}}(0, \mathbf{x}_{-s})] = \frac{v_s}{\theta_s}\left[\theta_s \sum_{t \in S_j : x_t = m} \theta_t - \frac{\theta_s(p_i^m - d_s^m)}{v_s}\right]$$

$$= \quad v_s \sum_{t \in S_j : x_t = m} \theta_t + d_s^m - p_i^m - 0 = u_s(m, \mathbf{x}_{-s}, \mathbf{p}) - u_s(0, \mathbf{x}_{-s}, \mathbf{p}). \quad \blacksquare$$

When there are multiple platforms, some agents may only have access to certain (but not all) platforms, i.e., $X_s \subseteq \{0, , \ldots, M\}$ for each s. Nevertheless, observe from Definition 1 that even if some agents' action sets X_s are shrunk, every subgame remains a weighted potential game with the same potential function (now defined on a smaller set of action profiles).

If platforms can charge each agent a different price $p_s^m \in \mathbb{R}$, every subgame remains a weighted potential game with p_i^m replaced by p_s^m. If they can only partially price discriminate agents, then there are some constraints on their prices p_s^m. When $M = 1$ and the platform can perfectly price discriminate agents, the model is a special case of Chan's (2021) multi-agent contracting model, in which the monopoly platform is the "principal" who contracts with multiple agents. Proposition 1 of that paper implies that the platform's optimal pricing strategy under potential maximization is

$$p_s^* = v_s \sum_{t \in S_j : \frac{v_t}{\theta_t} < \frac{v_s}{\theta_s}} \theta_t + d_s \quad \text{for all } s \in S_i \ (i, j = 1, 2;\ i \neq j),$$

assuming (for expositional convenience) that the platform wants to attract every agent and $\frac{v_s}{\theta_s} \neq \frac{v_t}{\theta_t}$ for all $s \in S_i$ and $t \in S_j$.

Same-Side Network Effects

The baseline model is generalized such that the payoff of a side-i agent from joining the platform is

$$u_i(n_i, n_j, p_i) = v_i n_j + f_i(n_i) - p_i,$$

where $f_i : \{1, \dots, N_i\} \to \mathbb{R}$ measures the same-side network effects on side i. If f_i is a constant function, it reduces to a stand-alone benefit/cost of joining the platform. We can verify that every subgame remains a weighted potential game with the potential function

$$\Phi_{\mathbf{p}}(n_1, n_2) = n_1 n_2 + \frac{1}{v_1}\sum_{k=1}^{n_1} f_1(k) + \frac{1}{v_2}\sum_{k=1}^{n_2} f_2(k) - \frac{p_1}{v_1} n_1 - \frac{p_2}{v_2} n_2.$$

If f_1 and f_2 are positive and increasing, the platform would like to attract all agents. Its profit maximization problem under potential maximization becomes

$$\max_{p_1, p_2 \geq 0} (p_1 - c_1)N_1 + (p_2 - c_2)N_2 \quad \text{s.t.} \quad (N_1, N_2) \in \arg\max \Phi_{\mathbf{p}}(n_1, n_2). \tag{B.5}$$

Given that cross-side and same-side network effects are both positive and increasing, there are only four possible equilibria in stage 2: (i) all agents join the platform, (ii) no one joins the platform, (iii) only all side-1 agents join the platform, and (iv) only all side-2 agents join the platform. Given that the potential maximizer is the equilibrium with the highest potential, (B.5) is simplified as follows:

$$\begin{aligned}
\max_{p_1, p_2 \geq 0} p_1 N_1 + p_2 N_2 \quad \text{s.t.} \quad \frac{p_1}{v_1} N_1 + \frac{p_2}{v_2} N_2 &\leq N_1 N_2 + \frac{1}{v_1}\sum_{k=1}^{N_1} f_1(k) + \frac{1}{v_2}\sum_{k=1}^{N_2} f_2(k), \\
\frac{p_1}{v_1} N_1 &\leq N_1 N_2 + \frac{1}{v_1}\sum_{k=1}^{N_1} f_1(k), \\
\frac{p_2}{v_2} N_2 &\leq N_1 N_2 + \frac{1}{v_2}\sum_{k=1}^{N_2} f_2(k).
\end{aligned}$$

The solution to this simple linear program is given as follows.

Proposition 5 *Suppose $v_1 < v_2$. Under potential maximization, the platform sets*

$$p_1^* = \frac{1}{N_1}\sum_{k=1}^{N_1} f_1(k), \quad p_2^* = v_2 N_1 + \frac{1}{N_2}\sum_{k=1}^{N_2} f_2(k).$$

Similar to the baseline model, the platform always divides and conquers. Moreover, the divide/conquer side does not depend on same-side network effects f_1 and f_2. Relative to the previous model, the platform marks up the price on each side by $\frac{1}{N_i}\sum_{k=1}^{N_i} f_i(k)$. Still, all agents are better off because the additional benefit $f_i(N_i)$ each side-i agent derives in equilibrium is greater than the price markup.

General Temporal Structure

The baseline model is modified as follows. Agents on each side are divided into two groups, indexed by I and II, and there are N_i^I group-I side-i agents and N_i^{II} group-II side-i agents ($N_i^I + N_i^{II} = N_i$). The game now has four stages. In stage 1, the platform sets $(p_1^I, p_2^I) \in \mathbb{R}^2$ to group-I agents. In stage 2, all group-I agents simultaneously decide whether to join the platform. In stage 3, the platform sets $(p_1^{II}, p_2^{II}) \in \mathbb{R}^2$ to group-II agents. In stage 4, all group-II agents simultaneously decide whether to join the platform. The history of play is common knowledge, and payoffs are realized at the end of stage 4. This model reduces to the baseline model if $N_1^I = N_2^I = 0$ or $N_1^{II} = N_2^{II} = 0$. Note that the analysis can be easily extended to situations where agents are partitioned into more than two groups.

This model is solved backwards. In stage 3, the numbers of group-I participants n_1^I and n_2^I are determined. Hence, for a side-i agent from group II, it is as if there is a stand-alone benefit of $v_i n_j^I$ from joining the platform. Thus, the platform's optimal pricing strategy in stage 3 is given by Proposition 5 in Appendix B with $f_i(\cdot)$ and N_i replaced by $v_i n_j^I$ and N_i^{II} respectively ($v_1 < v_2$):

$$p_1^{II*} = v_1 n_2^I, \quad p_2^{II*} = v_2(n_1^I + N_1^{II}).$$

In stage 2, all group-I agents anticipate all group-II agents will join the platform subsequently. Hence, for a side-i agent from group I, it is as if there is a stand-alone benefit of $v_i N_j^{II}$ from joining the platform. Thus, the platform's optimal pricing strategy in stage 1 is, again, given by Proposition 5:

$$p_1^{I*} = v_1 N_2^{II}, \quad p_2^{I*} = v_2(N_1^I + N_1^{II}) = v_2 N_1.$$

All agents join the platform in equilibrium (i.e., $n_i^I = N_i^I$), and thus its equilibrium profit is

$$\pi^* = v_1(N_1^I N_2^{II} + N_1^{II} N_2^I) + v_2 N_1 N_2 - c_1 N_1 - c_2 N_2.$$

Similar to the baseline model, the platform extracts all side-2 agents' surplus when $v_1 < v_2$. It now also extracts some surplus from side 1 because agents face less strategic uncertainty in the current framework. The surplus extracted from side 1 depends on the number of agents N_i^I and N_i^{II} in each group. If the platform can freely choose any numbers of group-I agents $(N_1^I, N_2^I) \in \{0, \ldots, N_1\} \times \{0, \ldots, N_2\}$, it will choose either $(N_1, 0)$ or $(0, N_2)$; it extracts all agents' surplus in either case. Suppose instead the platform faces a constraint: $N_1^I + N_2^I \leq n$, i.e., it can only approach at most $n \leq \min\{N_1, N_2\}$ agents in advance. The optimal choice is to approach n side-i agents whenever $N_i < N_j$ regardless of whether v_i or v_j is larger. Interestingly, the platform makes the same optimal choice if instead it can only delay n agents and approach them in stage 3.

Appendix C

Appendix for Chapter 3

Monopolist's Optimal Price under Favorable Expectations

I prove for the model in Section 3.3. The example in Section 3.2 is a special case with $v(i,q) = v$ and $\alpha(\sum_j a_j, q) = \sum_j a_j - 1$. Suppose the monopolist wants to attract $n \leq N$ consumers. For a fixed quality q, the highest possible price it can charge is

$$p(q) = v(n,q)\alpha\,(n,q) + \beta(n,q),$$

and only consumers 1 to n will buy the good under favorable expectations. Hence, from (3.1), its profit is

$$\pi((\underbrace{1,...,1}_{n\text{ “1”s}},\underbrace{0,...,0}_{N-n\text{ “0”s}}), p(q), q) = n(v(n,q)\alpha\,(n,q) + \beta(n,q)) - c(q).$$

Under B4, this function is increasing in n. Therefore, the monopolist optimally sets the price $p^*(q)$ according to (3.8) and attracts all consumers.

Proof of Lemmas 4 and 7

Here, I prove Lemma 7. The proof of Lemma 4 is a special case with $v(i,q) = v$ and $\alpha(\sum_j a_j, q) = \sum_j a_j - 1$. First, I show that the unique equilibrium (and thus the potential maximizer) is all consumers buying the good when $p < \beta(1,q)$, and no one buying the good when $p > v(N,q)\alpha(N,q) + \beta(N,q)$. When $p < \beta(1,q)$, consumer 1 has a dominant strategy to buy the good. Given consumer 1's dominant strategy, consumer 2 has an (iterated) dominant strategy to buy the good as well because

$$v(2,q)\alpha\,(2,q)+\beta(2,q)>\beta(1,q)>p,\quad \text{(by B4)}$$

and so on. Therefore, all consumers will buy the good as a dominant-solvable equilibrium. By the same token, we can easily verify that the unique (and dominant-solvable) equilibrium is no one buying the good when $p > v(N,q)\alpha(N,q)+\beta(N,q)$.

It remains to identify the potential maximizer when $\beta(1,q) \le p \le v(N,q)\alpha(N,q) + \beta(N,q)$, in which there are two equilibria. By (3.10), their respective potentials are

$$\Phi(\mathbf{1},p,q)=\sum_i \alpha(i,q)+\sum_i \frac{\beta(i,q)}{v(i,q)}-p\sum_i \frac{1}{v(i,q)},\quad \Phi(\mathbf{0},p,q)=0.$$

For these subgames, the potential maximizer is all consumers buying the good if and only if

$$p \le \hat{p}(q) \equiv \frac{\sum_i \alpha(i,q)+\sum_i \frac{\beta(i,q)}{v(i,q)}}{\sum_i \frac{1}{v(i,q)}}.$$

Last, I show that

$$\beta(1,q)<\hat{p}(q)<v(N,q)\alpha(N,q)+\beta(N,q), \tag{C.1}$$

and thus $\hat{p}(q)$ is indeed the cutoff determining which equilibrium is the potential maximizer. By B4, we have

$$\beta(1,q)<\cdots<v(i,q)\alpha(i,q)+\beta(i,q)<\cdots<v(N,q)\alpha(N,q)+\beta(N,q).$$

This implies for all $i \in \{2,\ldots,N-1\}$,

$$\frac{\beta(1,q)}{v(i,q)}<\alpha(i,q)+\frac{\beta(i,q)}{v(i,q)}<\frac{v(N,q)\alpha(N,q)+\beta(N,q)}{v(i,q)}.$$

Therefore,

$$\beta(1,q)\sum_i \frac{1}{v(i,q)}<\sum_i \alpha(i,q)+\sum_i \frac{\beta(i,q)}{v(i,q)}<(v(N,q)\alpha(N,q)+\beta(N,q))\sum_i \frac{1}{v(i,q)}.$$

Thus, (C.1) is true. Hence, we have identified the potential maximizer for all subgames, and it is summarized by Lemma 7.

Multidimensional Quality and the Generalized Cost Function

This appendix extends the model in Section 3.3 in two aspects. First, the payoff of consumer i from buying the good is generalized to

$$u_i(a_i = 1, \mathbf{a}_{-i}, p, \mathbf{q}) = v(i, \mathbf{q}) \cdot \alpha(\sum_j a_j, \mathbf{q}) + \beta(i, \mathbf{q}) - p,$$

where $\mathbf{q} \in \mathbb{R}_+^M$ is the M-dimensional quality of the good. Second, the monopolist's payoff is generalized to

$$\pi(\mathbf{a}, p, \mathbf{q}) = p \sum_i a_i - c(\sum_i a_i, \mathbf{q}),$$

where the generalized cost function $c : \{0, \ldots, N\} \times \mathbb{R}_+^M \to \mathbb{R}_+$ increases with the number of buyers.

B1–B4 are naturally modified as follows:

B1'. for all $i \in \{0, \ldots, N\}$, $c(i, \mathbf{q})$ is increasing, sufficiently convex, and differentiable in $\mathbf{q}$, and $\lim_{\mathbf{q} \to \mathbf{0}} \frac{\partial c(i,\mathbf{q})}{\partial \mathbf{q}} = \mathbf{0}$;

B2'. for all $i \in I$, $v(i, \mathbf{q})$, $\alpha(i, \mathbf{q})$, and $\beta(i, \mathbf{q})$ are increasing and differentiable in $\mathbf{q}$;

B3'. for all $\mathbf{q} \in \mathbb{R}_+^M$, $v(i, \mathbf{q})$ and $\beta(i, \mathbf{q})$ are decreasing in i;

B4'. for all $\mathbf{q} \in \mathbb{R}_+^M$, $\alpha(1, \mathbf{q}) = 0$, $c(1, \mathbf{q}) - c(0, \mathbf{q}) \leq \beta(1, \mathbf{q})$, and $v(i, \mathbf{q})\alpha(i, \mathbf{q}) + \beta(i, \mathbf{q}) - c(i, \mathbf{q})$ is strictly increasing in i.

B4' takes into account the cost function and assumes the marginal cost of production is small relative to the marginal network benefit of additional buyers. Thus, both the social planner and the monopolist always want to serve all consumers in the subsequent analysis.

I now discuss the analysis of this more general model. For any fixed quality profile $\mathbf{q} \in \mathbb{R}_+^M$, the analysis of the monopolist's price is almost identical to that of Section 3.3. The only difference is that we now need to take account of the marginal cost of production, which is assumed (in B4') to be relatively small. Thus, the monopolist's optimal pricing strategy is exactly the same as that in Section 3.3 (with q replaced by $\mathbf{q}$).

When determining the socially optimal and equilibrium quality profiles, instead of having one first-order condition, there are now M first-order conditions, one for each dimension of the quality. Nevertheless, each first-order condition takes exactly the same form as that in Section 3.3 (with $c(q)$ replaced by $c(N, \mathbf{q})$). Therefore, all insights developed in Section 3.3 carry over to this more general model.

Integer Solutions for the Number of Consumers

Let $\Delta f(N) \equiv f(N) - f(N-1)$ denote the (backward) difference of a function $f(N)$. The socially optimal number of consumers N^{FB} is uniquely given by

$$\begin{aligned}
\underbrace{\Delta c(N^{FB})}_{\text{marginal cost}} &\leq \underbrace{v(N^{FB})\alpha(N^{FB}) + \beta(N^{FB})}_{\text{marginal consumer's benefit}} + \underbrace{\Delta\alpha(N^{FB}) \sum_{i=1}^{N^{FB}-1} v(i)}_{\text{marginal benefits to others}}, \\
\Delta c(N^{FB}+1) &\geq v(N^{FB}+1)\alpha(N^{FB}+1) + \beta(N^{FB}+1) + \Delta\alpha(N^{FB}+1) \sum_{i=1}^{N^{FB}} v(i).
\end{aligned}$$

The equilibrium number of consumers N^* under favorable expectations is uniquely given by

$$\begin{aligned}
\Delta c(N^*) &\leq \underbrace{v(N^*)\alpha(N^*) + \beta(N^*)}_{\text{marginal consumer's benefit}} + \Delta\alpha(N^*)\underbrace{(N^*-1)v(N^*)}_{\text{Spence distortion}} \\
&+ \underbrace{(N^*-1)(\alpha(N^*-1)\Delta v(N^*) + \Delta\beta(N^*))}_{\text{market power distortion}}, \\
\Delta c(N^*+1) &\geq v(N^*+1)\alpha(N^*+1) + \beta(N^*+1) + \Delta\alpha(N^*+1)N^* v(N^*+1) \\
&+ N^*(\alpha(N^*)\Delta v(N^*+1) + \Delta\beta(N^*+1)).
\end{aligned}$$

The equilibrium number of consumers N^* under unfavorable expectations is uniquely given by

$$\Delta c(N^*) \leq \beta(1) \leq \Delta c(N^*+1).$$

The equilibrium number of consumers N^* under potential maximization is more complex, but we can characterize it in the same way as above, and thus omitted.

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